

## **ADMINISTRACIÓN DE PORTAFOLIOS A TRAVÉS DE OPERADORES HEAVY MOVING AVERAGES**

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### **Resumen**

Este artículo utiliza diferentes operadores de media móvil para innovar en la medición de la administración de portafolio. Entre los operadores que han utilizado se encuentran los operadores Heavy Ordered Weighted Moving Average (HOWMA) y el operador Induced HOWMA (IHOWMA). La principal ventaja de estos operadores es que pueden incluir en la misma formulación los datos históricos a través de las medias móviles y la experiencia y el conocimiento del responsable de la toma de decisiones con el vector de ponderación e inducido. Se desarrolló una aplicación para seleccionar en qué tipo de producto invertir, teniendo en cuenta las ventas históricas y el margen de utilidad, presentando una metodología original para incluir en el proceso de toma de decisiones.

**Palabras clave:** Medias móviles, operadores OWA, administración de portafolio.

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## **PORTFOLIO MANAGEMENT THROUGH HEAVY MOVING AVERAGE OPERATORS**

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### **Abstract**

This paper uses different moving average operators in a way to innovate the portfolio management measurement. Among the operators that has been used are the heavy ordered weighted moving average (HOWMA) operator and the induced HOWMA (IHOWMA) operator. The main advantage of these operators is that can include in the same formulation the historical data through the moving averages and the expertise and knowledge of the decision maker with the weighting and induced vector. An application for selection in which type of product to invest was developed, taking into consideration historical sales and profit margin, presenting an original methodology to include in the decision-making process.

**Keywords:** heavy moving averages, OWA operator, portfolio management.

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## 1. INTRODUCTION

Innovation management has been studied by several authors through the decades because there is a common idea that is the driver for competitiveness because some of its effects are increase in sales, market share, productivity and efficiency (Porter, 1990; Ernst, 2002; Keupp *et al.*, 2012).

One of the main concerns in innovation management is the measurement since this determines if the resources used by the company are justified, but the process to quantify and evaluate the effects of the innovation in the company is very complex (Frenkel *et al.*, 2000; Gimbert *et al.*, 2010).

One of the frameworks to measure innovation in the organizations is presented by Adams *et al.* (2006) and is extended by Alfaro-Garcia *et al.* (2017), this approach proposes seven key innovation measurement areas: 1. Innovation strategy, 2. Knowledge management, 3. Project management, 4. Portfolio management, 5. Internal drivers, 6. Organization and structure and 7. External drivers.

This paper focus in portfolio management that is the extent to which firms base their operations on systematized processes that are guided by clear criteria (Alfaro-Garcia *et al.*, 2017; Hall and Nauda, 1990), in this sense, generate processes where the knowledge and the expertise of the decision makers can be added will help to generate different scenarios that must be considered in the selection process of the alternatives.

A common aggregation method is the ordered weighted averaging (OWA) operator introduced by Yager (1988). Since then, the OWA operator has been used in a lot of applications (Kacprzyk & Zadrozny, 2009; Merigó & Gil-Lafuente, 2010; Xu & Da, 2003) and extended under a wide range of frameworks (Merigó, 2010).

This paper focuses on different extension of heavy moving average operators such as the heavy ordered weighted moving average (HOWMA) operator and the induced HOWMA (IHOWMA) operator. These operators used as base three classical aggregation techniques that are the induced ordered weighted average (IOWA) operator, the

heavy ordered weighted average (HOWA) operator and the moving averages (MA) operator. The IOWA operator was introduced by Yager & Filev (1999). In the IOWA operator, the reordering step is not developed with the values of the arguments. Instead is induced by another mechanism such that the ordered position of the arguments depending upon the values of their associated order-inducing variables.

The HOWA operator was introduced by Yager (2002) and provides a parameterized family of aggregation operators that includes among other, the minimum, the OWA operator and the total operator. The main advantage of this operator is that it provides a wider class of aggregation operator by allowing the weighting vector to range between the OWA and the total operator. This operator has been studied by using fuzzy measures (Yager, 2003) and fuzzy numbers (Merigó & Casanova, 2008). Also, more extensions have been developed by Merigó (2008).

The aim of this paper is to analyze the use heavy moving average operators in portfolio management innovation, specifically in the selection of a product to invest based in the historical sales and profitability. The main advantage of the heavy moving average operators is that can include the expectations and knowledge of the market of the decision makers.

The remainder of the paper is organized as follows. In Section 2 we review the OWA operator and some previous approaches. Section 3 the specific cases of the IHOWMA and HOWMA operator is presented. Section 4 presents the use of the heavy moving average operator in portfolio management innovation and Section 5 summarizes the main conclusions of the paper.

## **2. PRELIMINARIES**

In this section, we briefly review some basic concepts to be used throughout the paper. We analyze the OWA operator, the heavy aggregation operators, the induced aggregation operators and the moving averages.

### 2.1. OWA operator

The OWA operator was introduced by Yager (1988). It provides a parameterized family of aggregation operators which have been used in many applications (Merigo, 2010; Xu & Da, 2003). In the following, we provide a definition of the OWA operator as introduced by Yager (1988).

**Definition 1.** An OWA operator of dimension  $n$  is an application  $F: R^n \rightarrow R$  with an associated weight vector  $w = [w_1, w_2, \dots, w_n]^T$  so that  $w_j \in [0, 1]$ ,  $1 \leq i \leq n$  and

$$\sum_{i=1}^n w_i = 1, \tag{1}$$

where

$$F(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j, \tag{2}$$

being  $b_j$  the  $j$ th largest element of the collection  $a_1, a_2, \dots, a_n$ .

Note that we can distinguish between the descending OWA (DOWA) operator and the ascending OWA (AOWA) operator. The weights of these operators are related by  $w_j = w_{n-j+1}^*$ , where  $w_j$  is the  $j$ th weight of the DOWA operator and  $w_{n-j+1}^*$  the  $j$ th weight of the AOWA operator.

The heavy OWA (HOWA) operator introduced by Yager (2002) is an extension of the OWA operator. In this case, the difference with the OWA operator is that the sum of the weights is allowed to be between 1 and  $n$ , instead of being restricted to sum up to 1. In the following, we provide a definition of the HOWMA operator suggested by Yager (2002).

**Definition 2.** A heavy aggregation operator, is an extension to OWA operator that allows the weight vector goes up to  $n$ . So, a HOWA operator is an application  $R^n \rightarrow R$  which are associated to a weight vector  $w$  which  $w_j \in [0, 1]$  and  $1 \leq \sum_{j=1}^n w_j \leq n$ , so that

$$HOWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j \quad (3)$$

being  $b_j$  the  $j$ th largest element of the collection  $a_1, a_2, \dots, a_n$ .

The HOWA operator is monotonic and commutative. Note that the HOWA operator is not bounded by the minimum and the maximum. In this case, it is bounded by the minimum and the total operator which represents the sum of all arguments. We can distinguish between the descending HOWA (DHOWA) operator and the ascending HOWA (AHOWA) operator.

The Induced OWA operator (IOWA) operator was introduced by Yager and Filev (1999) as an extension of the OWA operator. Its main difference is that the reordering step is not developed with the values of the arguments  $a_i$ . In this case, the reordering step is developed with order inducing variables. The IOWA operator can be defined as follows.

**Definition 3.** An IOWA operator of dimension  $n$  is an application  $IOWA: R^n \times R^n \rightarrow R$  that has a weighting vector associated  $W$  of dimension  $n$  where the sum of the weights is 1 and  $w_j \in [0,1]$ , where a induced set of ordering variables are included  $(u_i)$  so the formula is

$$IOWA(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^n w_j b_j, \quad (4)$$

where  $b_j$  is the  $a_i$  value of the OWA pair  $\langle u_i, a_i \rangle$  having the  $j$ th largest  $u_i$ .  $u_i$  is the order inducing variable and  $a_i$  is the argument variable.

From a generalized perspective of the reordering step, we can distinguish between the descending IOWA (DIOWA) operator and the ascending IOWA (AIOWA) operator. The weight of these operators are related by  $w_j = w_{n-j+1}^*$ , where  $w_j$  is the  $j$ th weight of the DIOWA and  $w_{n-j+1}^*$  the  $j$ th weight of the AIOWA operator.

The IOWA operator is an averaging operator. This is a reflection of the fact that the operator is monotonic, commutative, bounded and

idempotent both for the DIOWA and the AOWA operator. Note that the OWA operator is obtained when  $u_i = a_i$ , for all  $i$  [12, 18, 20].

**2.2 Moving averages**

A moving average is a usual average that moves toward some part of the whole sample. More generally, the moving average can be seen as a moving aggregation operator. This method is known for solving time-series smoothing problems and has been applied extensively in economics and statistics (Evans, 2002). Its main advantage in this context is the possibility of forecasting future results based on the historical data. The moving average, according to Kenney and Keeping (1962) can be defined as follows.

**Definition 4.** The moving averages are defined as a sequence given  $\{a_i\}_{i=1}^N$ , where a moving average  $n$  is a new sequence  $\{s_i\}_{i=1}^{N-n+1}$  defined from  $a_i$  taking the arithmetic mean of the sequence of  $n$  terms, such that

$$s_i = \frac{1}{n} \sum_{j=i}^{i+n-1} a_j, \tag{5}$$

Another extension using moving averages is when we combine it with the OWA operator generating the Ordered Weighted Moving Average (OWMA), Merigo and Yager (2013) defined it as follows.

**Definition 5.** An Ordered Weighted Moving Average (OWMA) of dimension  $m$  is a mapping  $OWMA: R^m \rightarrow R$  that has an associated weighting vector  $W$  of dimension  $m$  with  $W = \sum_{j=1+t}^{m+t} w_j = 1$  and  $w_j \in [0,1]$ , such that:

$$OWMA (a_{1+t}, a_{2+t}, \dots, a_{m+t}) = \sum_{j=1+t}^{m+t} w_j b_j, \tag{6}$$

where  $b_j$  is the  $j$ th largest argument of the  $a_i$ ,  $m$  is the total number of arguments considered from the whole sample and  $t$  indicates the movement done in the average from the initial analysis.

### 3. HEAVY MOVING AVERAGE OPERATORS

The induced heavy ordered weighted moving average (IHOWMA) operator is presented by León-Castro *et al.* (2017) and it consists in the inclusion of induced weights in the classical formulation of the last one. In general, this operator includes the HOWMA operator as a particular case. It can be defined as follows.

**Definition 6.** A IHOWMA operator is defined as a given sequence  $\{a_i\}_{i=1}^N$ , where you get a new sequence  $\{s_i\}_{i=1}^{N-m+1}$  which is multiplied by a heavy weighting vector, such that

$$IHOWMA(\langle u_1, a_{1+t} \rangle, \langle u_2, a_{2+t} \rangle, \dots, \langle u_n, a_{m+t} \rangle) = \sum_{j=1+t}^{m+t} w_j b_j, \quad (7)$$

where  $b_j$  is  $j$ th element that has the largest value of  $u_i$ ,  $u_i$  is the order inducing variables, and  $W$  is an associated weighting vector of dimension  $m$  with  $W: 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$  and  $w_i \in [0,1]$ . Observe that here we can also expand the weighting vector from  $-\infty$  to  $\infty$ . Thus, the weighting vector  $w$  becomes  $-\infty \leq \sum_{j=1}^n w_j \leq \infty$ .

When analyzing the magnitude of the weighting vector  $|W|$  for the IHOWMA operator, we can use the same methodology used by Yager (2002) for the HOWA operator. As this future does not depend upon the reordering of the arguments, the formulation is the same as for the HOWA operator. Thus, it is defined as  $\beta(W) = (|W| - 1)/(n - 1)$ . Since  $|W| \in [1, n]$ , then  $\beta \in [0,1]$ . That is why if  $\beta = 1$ , we get the total operator and if  $\beta = 0$ , we get the usual Moving Average.

Note that if  $u_i = 1/m$  for all  $i$ , the IHOWMA becomes the Heavy Ordered Weighted Moving Average (HOWMA), that can be defined as follows (Leon-Castro *et al.*, 2016).

**Definition 9.** A HOWMA operator is defined as a given sequence  $\{a_i\}_{i=1}^N$ , where you get a new sequence  $\{s_i\}_{i=1}^{N-n+1}$  which is multiplied by a heavy weighting vector, such that

$$HOWMA(s_i) = \sum_{j=1+t}^{m+t} w_j b_j, \quad (10)$$

where  $b_j$  is the  $j$ th largest element of the collection  $a_1, a_2, \dots, a_n$ , and  $W$  is an associated weighting vector of dimension  $m$  with  $W: 1 \leq \sum_{i=1+t}^{m+t} w_i \leq n$  and  $w_i \in [0,1]$ . Observe that here we can also expand the weighting vector from  $-\infty$  to  $\infty$ . Thus, the weighting vector  $w$  becomes  $-\infty \leq \sum_{j=1}^n w_j \leq \infty$ .

#### **4. INDUCED HEAVY MOVING AVERAGE OPERATORS IN PORTFOLIO MANAGEMENT**

Portfolio management is vital for a successful product innovation and is needed in order to know in which market, product or technology the enterprises will invest their resources (Cooper *et al.*, 1999). Also, when the company has multiple projects the use of a methodology that helps to achieve the best allocation for resources is needed in order to generate more benefits. These methodologies are needed to be agile ones that can be adapted to the rapid changes of the markets (Stettina & Hörz, 2015). In the present we consider three different categories of products

$a_1$  *Grocery*

$a_2$  *Medicine*

$a_3$  *Perishable*

$a_2$  *Stationary*

The historical sales information for each alternative is as follows (See table 1).

Date	Grocery	Medicine	Perishable	Stationary
01-16	63.5	36.5	43.2	61.3
02-16	72.1	38.6	38.9	67.5
03-16	45.3	40.5	46.8	60.2
04-16	39.8	46.1	39.4	58.9
05-16	55.4	44.6	41.2	61.3
06-16	62.4	55.6	50.2	57.8
07-16	57.6	48.9	48.3	70.2
08-16	49.8	34.5	43.2	71.5
09-16	72.4	36.9	47.5	69.8
10-16	58.3	42.5	44.6	65.4
11-16	47.2	35.4	39.7	60.8
12-16	74.8	36.2	41.5	59.8

Table 1. Historical sales of the alternatives (Sales are in thousands)

Note that  $n = 12$  because the decision maker wants to take into account all the information available, the ordered inducing variables  $U = (2,3,8,9,5,6,7,4,12,10,11,1)$  for all the alternatives and the weighted vectors are  $W_{a_1} = (0.05,0.05,0.20,0.10,0.10,0.05,0.05,0.10,0.10,0.05,0.05,0.20) = 1.10$  and  $W_{a_2} = (0.10,0.05,0.05,0.05,0.10,0.15,0.15,0.05,0.05,0.10,0.05,0.20) = 1.10$ . All the vectors are higher than 1 because the decision maker believes that 2017 will be a better year than 2016.

Using the above information, the operators Moving Average (MA), HOWMA and IHOWMA are applied to generate various selection scenarios (See table 2).

	<i>Grocery</i>	<i>Medicine</i>	Perishable	Stationary
MA	58.21	41.35	43.71	63.71
HOWMA	58.20	44.10	46.44	67.95
IHOWMA	64.04	47.02	46.14	66.60

Table 2. Results using aggregation operators

The decision maker also wants to include the profit margin, that are  $a_1 = 20\%$ ,  $a_2 = 28\%$ ,  $a_3 = 25\%$  and  $a_4 = 18\%$ . The results are as follow (See table 3)

	<i>Grocery</i>	<i>Medicine</i>	Perishable	Stationary
MA	11.64	11.58	10.93	11.47
HOWMA	11.63	12.34	11.61	12.23
IHOWMA	12.80	13.16	11.53	11.99

Table 3. Results using aggregation operators and profit margin

Finally, let us rank the result according to each of the methods and information used in the analysis (See table 3-4).

Method	Ranking
Moving Average	$a_4 > a_1 > a_3 > a_2$
HOWMA	$a_4 > a_1 > a_3 > a_2$
IHOWMA	$a_4 > a_1 > a_3 > a_2$

Table 4. Ranking of the alternatives using aggregation operators

Method	Ranking
Moving Average	$a_1 > a_2 > a_4 > a_3$
HOWMA	$a_2 > a_1 > a_4 > a_3$
IHOWMA	$a_2 > a_1 > a_4 > a_3$

Table 5. Ranking of the alternatives using aggregation operators and profit margin

Note that using the IHOWMA operator the order of the alternatives changes, this is because the inclusion of the induced weighted vector instead of the regular one. In this sense, the ranking of alternatives includes more information about the decision maker, instead of only taking into account the historical data, with the use of the IHOWMA operator the knowledge, expectations and characteristics of the decision maker can be added and make the results more complex and specialized.

Also, it is important to note the difference when we only take into account the sales where always  $a_4$  is the best option, but when we include the profit margin the results vary from  $a_1$  to  $a_2$  depending on the operator that we use. This analysis is important to portfolio management because sometimes the company has products that make bigger sales and other that has more profit. In this sense, if the enterprise wants to increase their sales in order to increase their market share or wants to increase their profitability the decision to invest in which product will change.

## **5. CONCLUSIONS**

This paper uses different moving average operators in a way to innovate the portfolio management and generate new scenarios that the decision maker has to take into account in order to have a clear vision of where the company wants to go. The main advantage of using heavy moving average operators is that it takes into account the historical data and the expertise and knowledge of the decision maker in the same formulation. It is also important to note that the HOWMA and IHOWMA operator includes a wide range of particular cases that can be applied into different problems.

As can be seen in the results, it is important to take into account different organizational factors in order to make the better decision, such as expectations of sales, profit margins, market share, cost and many more. That is why we presented an adaptatively and agile methodology that can be used in order to forecast different scenarios that will help the organization to have a clearer view of the future and make a more efficient decision.

In future research, we expect to develop further the operators by using more complex formulations such as quasi-arithmetic means, the use of fuzzy numbers, interval numbers or taking into account a group decision making process, prioritized aggregation operators or expertons (Perez-Arellano *et al.*, 2017; Kaufmann, 1988; Merigó *et al.*, 2014)

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