ISSN 2451-5728

Serie Documentos de Trabajo del IIEP

Nº 44 - Noviembre de 2019

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Ministerio de Ciencia, Tecnología e Innovación Productiva
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Tel +54 11 5983-1420

http://www.conicet.gov.ar/

ISSN 2451-5728

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Desarrollo editorial: Ed. Hebe Dato

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MODELO DE COMPONENTES DE ERRORES PARA REDES: ESPECIFICACIÓN Y CONTRASTES

ABSTRACT
This paper develops a subgraph network random effects error components for network data regression models. In particular, it allows for edge and triangle specific components, which serve as a basal model for modeling network effects. It then evaluates the potential effects of ignoring network effects in the estimation of the variance-covariance matrix. It also proposes consistent estimator of the variance components and Lagrange Multiplier tests for evaluating the appropriate model of random components in networks. Monte Carlo simulations show that the tests have good performance in finite samples. It applies the proposed tests to the Call interbank market in Argentina.

RESUMEN
Este trabajo desarrolla modelos de componentes de errores para regresiones con datos en redes. En particular, el modelo permite efectos específicos de links y triángulos, que sirven como una primera aproximación para modelar efectos de redes más complejos. Se evalúan las consecuencias de ignorar los efectos de redes sobre la estimación de la matriz de varianzas y covarianzas en modelos de regresión. Se proponen estimadores consistentes de los componentes de la varianza y contrastes de multiplicadores de Lagrange para evaluar el modelo correcto a ser usado. Simulaciones de Monte Carlo muestran una buena performance en muestras finitas. Se aplican los contrastes al mercado interbancario Call en Argentina.

Keywords: NETWORKS - CLUSTERS - MOULTON FACTOR
Palabras claves: REDES - CLUSTERS - FACTOR DE MOULTON

JEL Codes: C2 y C12

1 Introduction

Statistical inference when data are grouped into clusters is an important issue in empirical work, and failure to control for within-cluster correlation can lead to misleadingly small standard errors (see, e.g., the discussion in Cameron and Miller, 2015). This is especially important when using aggregate variables on micro units in which ordinary least-squares (OLS) standard errors are seriously underestimated. The seminal work of Moulton (1986, 1987, 1990) allows for a quantification of this potential pitfall, a fact that has been emphasized in chapter 8 of Angrist and Pischke (2009) textbook among many others (see Montes-Rojas, 2016).

A particular data structure related to cluster effects is that of networks. Matched data, where the interaction among agents is observed, are one type of such network data, where the information on who is in direct or indirect contact with whom matters. This has attracted a considerable attention with regards to spillover effects in education, production, financial markets, trade and many others. See Chandrasekhar (2016), de Paula (2017) and Graham (2019) for recent literature reviews.

Within a given network observations are not independent and the dependence structure is related to the network position of the observation. There is no obvious pattern to construct clusters or groups in this case. Network models differ from standard cluster ones in the heterogeneity of the groups which need to be defined ad-hoc within the network as there are no obvious way to group observations. The network structure also differs from the spatial case, as in the latter there is a natural embedding into some metric space (i.e., natural geometry). The most obvious type of intra-network correlation arises when we consider observations given by vertices or nodes that have a common edge or link. If we consider a link-specific effect, this would result in a specific factor that arises for linked nodes and not for others. Nodes that share a link might be correlated with each other.

We are mostly concerned with a linear regression model where observations are the nodes and with the correct estimation of the variance-covariance structure. Thus we explore error components structure where the components depend on local network features of the observations. In particular, for a given graph we construct the error components model by considering link- and triangle-specific effects. The main purpose of this exercise is that the empirical researcher starts from a standard variance-covariance structure (i.e., independent error components), and then tests sequentially for potential components' patterns (e.g., edges, triangles, diamonds, cliques, stars, etc.). In particular, this paper is concerned with the clustered characteristics of nodes and links.

There is a substantial theoretical and empirical research addressing why agents may prefer to have clustered links. Jackson, Barraquer, and Tan (2012) develop a simple model of favor exchange, in which being part of a group of three generates a payoff, whereas having a link or two friends who themselves are unlinked may generate no payoff. Moreover, correlation in links could allow agents to sustain cooperation that they may otherwise not be able to as in Bloch, Genicot, and Ray (2008) and Karlan, Mobius, Rosenblat, and Szeidl (2009). As noted in Chandrasekhar (2016) a simple extension of standard models to take into account these effects is to impose a triangle effect. Chandrasekhar and Jackson (2016) study random graphs based on subgraphs where the network is the union of these subgraphs. As in this literature, we develop the particular example of triangle effects, as different from node and link characteristics.

The developed error components model has interesting features.

First, contrary to the standard error components models, network effects will typically imply heteroskedastiticy. Take for instance the vertex&edge-only error components model where each vertex will have a vertex specific random component and an edge specific random component. Vertices that have one link are different from those that have two or more. The edge specific component will in fact generate a higher variance for vertices with more links.

Second, as with the Moulton factor, the key role is given by the joint consideration of the intra-network correlation of the error term(s) and the covariates. More formally, given an intra-network covariance structure of the error term and one of the covariates, the potential effect of misspecification in the variance-covariance of the estimators will depend on the sample intra-network covariance between the covariance factors of the error term and the covariates.

In most empirical settings, both covariance factors are positively correlated (i.e., a high correlation between two unobservables usually corresponds to a high correlation between the covariates), and thus this determines that the OLS estimator variance that do not consider the potential network effects will underestimate the true variance. In particular, in the special case of covariates with no intra-network correlation, the standard OLS variance is correct.

This paper differs from the literature in several ways.

First, many network econometrics related contributions focus on dyadic data structures where the unit of observation is the pair, i.e., the link, rather than the node (see, for instance, Krackhardt, 1988; Hoff, Raftery, and Handcock, 2002; Hoff, 2005). However, as noted by Chandrasekhar and Jackson (2016) that approach is not designed for identifying the incidence of particular patterns within network relationships. For regression models in which the node rather than the link is the unit of observation, the proposed subgraph network application is more useful.

Second, most of the linear regression network models using nodes as the unit of observations build upon spatial econometric models. The seminal contribution is Manski (1993), but there is a large literature on developing identification and estimation of network models with spatial-type structure (see, e.g., Kelejian and Prucha, 1999; Lee, 2007; Kelejian and Prucha, 2010;

Lee, Liu, and Lin, 2010). Spatial models have the advantage of estimating few parameters (i.e., the spatial autoregressive parameter). These models require a pre-specification of the adjacency matrix and a distance measure within the network topology, which may be subject to misspecification. Once this is given there is a fixed relation among the variance-covariance components that multiply the powers of adjacency matrix. The potential gains from using a subgraph network error component model rather than a spatial model would depend on the flexibility of adding additional components without restrictions on the variance-covriance structure. Alternatively, many network features can be modeled from imposing additional parameters on the powers of the adjacency matrix. Spatial econometric models are not specifically modeled for capturing subgraph structures that may affect the dependence among observations. We argue that our approach provides a more flexible model to account for that. Note, however, that the proposed analysis is a complement to spatial econometric models, and it does not intend to show it is better than those in a given dimension.

This paper is organized as follows. Section 2 develops the subgraph network random effects error components model. Section 3 presents simple consistent variance-covariance components estimators. Section 4 constructs specification tests and Section 5 evaluates the finite sample performance using Monte Carlo experiments. Section 6 applies the proposed tests to the interbank market in Argentina. Section 7 concludes.

2 Network error components model

2.1 Network definition and notation

Consider an undirected graph $G = (V, L)$ as a mathematical structure consisting of a set V of vertices (also commonly called nodes) and a set L of edges (also commonly called links). Unless otherwise specified the graph is undirected where elements of L are unordered pairs (i, s) of distinct vertices $(i, s) \in V \times V$. If the graph were directed where the elements of L are ordered pairs $(i, s) \in V \times V$. The number of vertices is $N = |V|$ and the number of edges is $M = |L|$. Without loss of generality, we will label the vertices simply with the integers $1, \ldots, N$, and the edges, $1, \ldots, M$. Note that $M \leq N(N-1)/2$ for undirected graphs (and $M \leq N(N-1)$ for directed ones). There are $2^{\binom{N}{2}}$ potential undirected networks.

For our purposes consider a set of triangles in undirected graphs as $Triangles = \{(i, s, r) \in V^3, i < s < r, (i, s), (s, r), (i, r) \in L^3\}$, the number of triangles is $T \leq N(N-1)(N-2)/6$. The set of triangles could be defined differently for directed graphs.

The fundamental connectivity of a graph G may be captured in an $N \times N$ binary adjacency matrix A with entries

$$a_{is} = \begin{cases} 1 & \text{if vertices } \{i, s\} \in L \\ 0 & \text{otherwise} \end{cases},$$

the edge-incidence matrix B, an $N \times M$ binary matrix with entries

$$b_{ij} = \begin{cases} 1 & \text{if vertex } i \text{ is incident to edge } j \\ 0 & \text{otherwise} \end{cases},$$

and the triangle incidence matrix C, an $N \times T$ binary matrix with entries

$$c_{ik} = \begin{cases} 1 & \text{if vertex } i \text{ is incident to triad } k \\ 0 & \text{otherwise} \end{cases}.$$

For an undirected network A is symmetric and we can define the vertices' degree $\{d_i\}_{i=1}^N$ which can be obtained by $diag(BB^\top)$, and vertices' triangles $\{t_i\}_{i=1}^N$ which can be obtained by $diag(CC^\top)$.

The definitions above correspond to unweighted networks. We could extend this to weighted networks by defining an $N \times N$ binary matrix w with entries

$$w_{is} = \begin{cases} w_{is} & \text{if vertices } \{i, s\} \in L \\ 0 & \text{otherwise} \end{cases}.$$

The matrices B and C need to be constructed accordingly.

2.2 Subgraph random effects in the undirected graph model

Consider the following assumption on the probability space.

Assumption 1:

Let $G \in \mathcal{G}_N$ be a space of graphs of size N and $x \in \mathcal{X}_N \in \mathbb{R}^{KN}$ the domain of K covariates, $\sigma(\mathcal{G}_N, \mathcal{X}_N)$ a σ-algebra in the sample space $(\mathcal{G}_N, \mathcal{X}_N)$, and $\mathcal{P}_N$ a probability space on the measurable space on $(\mathcal{G}_N, \mathcal{X}_N), \sigma(\mathcal{G}_N, \mathcal{X}_N)$. Then $[(\mathcal{G}_N, \mathcal{X}_N), \sigma(\mathcal{G}_N, \mathcal{X}_N), \mathcal{P}_N]$ form a probability space.

Now we consider a standard regression model for observations given by nodes within the network:

Assumption 2: Consider then the error components regression model for an unweighted undirected subgraph network structure,

$$y_i = x_i\beta + \varepsilon_i, \tag{1}$$

where the $i = 1, 2, ..., N$ observations are connected through a graph G as defined in Section 2.1.

This paper is concerned with the error structure of the regression model above:

Assumption 3:

Assume that

$$\varepsilon_i := \mathrm{E}[y_i - E(y_i \mid x, G)] = \nu_i + \sum_{j=1}^{M} b_{ij}\mu_j + \sum_{k=1}^{T} c_{ik}\delta_k, \tag{2}$$

where $\{b_{ij}\}$ and c_{ik} are the row vectors of matrices B and C derived from graph G, and ν, μ and δ are mutually independent random variables that satisfy: (i) $\forall_{i,j,t}$ $\mathrm{E}(\nu_i \mid x, G) = \mathrm{E}(\mu_{ij} \mid x, G) = \mathrm{E}(\delta_{ijt} \mid x, G) = 0$, (ii) $\forall_{i,j,t}$ $\mathrm{Var}(\nu_i \mid x, G) = \sigma^2_\nu$, $\mathrm{Var}(\mu_{ij} \mid x, G) = \sigma^2_\mu$, $\mathrm{Var}(\delta_{ijt} \mid x, G) = \sigma^2_\delta$.

Let ν, μ and δ be mutually independent random vectors of size N, M and T, respectively.

Correct mean specification: $\forall_{i,j,t}$ $\mathrm{E}(\nu_i \mid x, G) = \mathrm{E}(\mu_{ij} \mid x, G) = \mathrm{E}(\delta_{ijt} \mid x, G) = 0$.

Variance: $\forall_{i,j,t}$ $\mathrm{Var}(\nu_i \mid x, G) = \sigma^2_\nu$, $\mathrm{Var}(\mu_{ij} \mid x, G) = \sigma^2_\mu$, $\mathrm{Var}(\delta_{ijt} \mid x, G) = \sigma^2_\delta$.

The model can also be written as

$$\varepsilon_i = \nu_i + \sum_{i=1}^{N} \sum_{s>i}^{N} a_{is}\mu_{(is)} + \sum_{i=1}^{N} \sum_{s>i}^{N} \sum_{r>s}^{N} a_{is}a_{sr}a_{ir}\delta_{(isr)}, \tag{3}$$

where $\mu_{(is)}$ and $\delta_{(isr)}$ correspond to the common edge and triangle effects, respectively.

In matrix notation the model above can be written as $y = x\beta + \varepsilon$, where y and ε are $N \times 1$ vectors, x is $N \times K$ matrix, and β is a $K \times 1$ vector. Then consider

$$\varepsilon = \nu + B\mu + C\delta, \tag{4}$$

and

$$\begin{aligned} \Omega := \mathrm{E}[\varepsilon\varepsilon^\top \mid x, G] &= E[\nu\nu^\top + B\mu\mu^\top B^\top + C\delta\delta^\top C^\top \mid x, G] \\ &= \sigma^2_\nu I_N + \sigma^2_\mu BB^\top + \sigma^2_\delta CC^\top, \end{aligned} \tag{5}$$

where ν is a $N \times 1$ random vector, μ is a $M \times 1$ random vector, δ is a $T \times 1$ random vector.

Note that this model allows for the covariates x to be dependent on the network structure. Thus for instance, vertex-specific features such as network centrality (degree, betweenness, clustering, etc.) may be covariates of the model.

Consider the OLS estimator $\hat{\beta} = (x^\top x)^{-1} x^\top y$, and consider the goal of estimating $Var[\hat{\beta} \mid x, G]$. Given the assumptions of the model, then consider

$$Var[\hat{\beta} \mid x, G] = (x^\top x)^{-1}(x^\top \Omega x)(x^\top x)^{-1}. \tag{6}$$

Note that Ω acts as a selector and weighting matrix that selects which row and columns of x should be considered and weights them accordingly.

In the case with no network effects, defined as the vertex-only model,

$$\Omega_v = \sigma_\nu^2 I_N,$$

and thus only the xs that correspond to the same vertices i are considered. Thus

$$x^\top \Omega_v x = \sigma_\nu^2 \sum_{i=1}^{N} x_i x_i^\top.$$

The random-effects vertex&edge incidence model would have

$$\Omega_{ve} = \sigma_\nu^2 I_N + \sigma_\mu^2 B B^\top.$$

Thus

$$\begin{aligned} x^\top \Omega_{ve} x &= \sum_{i=1}^{N} (\sigma_\nu^2 + d_i \sigma_\mu^2) x_i x_i^\top + 2\sigma_\mu^2 \sum_{i=1}^{N-1} \sum_{s>i}^{N} a_{is} x_i x_s^\top. \\ &= \sum_{i=1}^{N} (\sigma_\nu^2 + d_i \sigma_\mu^2) x_i x_i^\top + 2\sigma_\mu^2 \sum_{i=1}^{N-1} \sum_{s>i}^{N} (\sum_{j=1}^{M} b_{ij} b_{sj}) x_i x_s^\top. \end{aligned}$$

Two things are important to notice from this variance-covariance. First, note that the model implies an heteroskedastic structure, where the diagonal elements are proportional to the degree d_i of each vertex. Second, the off-diagonal elements that have a role are those of vertices that have a common link, which in this case have a maximum of one. As in the case of the standard Moulton factor for one-way random effects error component models, the variance of the OLS estimators may be under estimated if the Xs are correlated in the same way as the errors. For this case, note that if $\sum_{i=1}^{N-1}\sum_{s>i}^{N}(\sum_{j=1}^{M} b_{ij}b_{sj})x_i x_s^\top$ is positive definite then a regression model that does not take into account the network structure will underestimate the true variance of $\hat{\beta}$.

The random-effects vertex&triangle incidence model would have

$$\Omega_{vt} = \sigma_\nu^2 I_N + \sigma_\delta^2 CC^\top.$$

Therefore, the variance-covariance matrix component is

$$\begin{aligned} x^\top \Omega_{vt} x &= \sum_{i=1}^{N}(\sigma_\nu^2 + t_i\sigma_\delta^2)x_i x_i^\top + 2\sigma_\delta^2 \sum_{i=1}^{N-2}\sum_{s>i}^{N-1}\sum_{r>s=1}^{N} a_{ir}a_{sr}a_{is}x_i x_s^\top \\ &= \sum_{i=1}^{N}(\sigma_\nu^2 + t_i\sigma_\delta^2)x_i x_i^\top + 2\sigma_\delta^2 \sum_{i=1}^{N-1}\sum_{s>i}^{N}(\sum_{k=1}^{T} c_{ik}c_{sk})x_i x_s^\top. \end{aligned}$$

In the same way as the vertex&edge model, this model has an heteroskedastic structure that depends on the number of triangles each vertex belongs to. Moreover the off-diagonal elements are proportional to the number of triangles each edge belongs to (maximum $N-2$). Moreover, we can have a Moulton factor-type analysis when comparing this model with the *iid* or vertex-only model.

Joining both models gives a general model with vertex, edge and triangle random components.

$$x^\top \Omega_{vet} x = \sum_{i=1}^{N}(\sigma_\nu^2 + d_i\sigma_\mu^2 + t_i\sigma_\delta^2)x_i x_i^\top + 2\sum_{i=1}^{N-1}\sum_{s>i}^{N}\left[\sigma_\mu^2 \sum_{j=1}^{M} b_{ij}b_{sj} + \sigma_\delta^2 \sum_{k=1}^{T} c_{ik}c_{sk}\right] x_i x_s^\top.$$

The results above can be easily extended to weighted networks where A is replaced by W, and the B and C matrices are also constructed using the weighted components. Note that for weighted networks the potential misspecification problems in estimating the variance-covariance components are likely to be more severe if $w_{is} \propto x_i x_s'$.

2.3 Comparison with spatial models

Consider now a simple spatially autocorrelated models, where the matrix A is related to distance, and for this case A is a symmetric contiguity matrix. See Anselin, Bera, Florax, and Yoon (1996) for a discussion and a comparison of different models.

The simplest model of spatial autocorrelation is captured by a simple autocorrelation parameter ρ such that

$$\varepsilon = \nu + \rho A \nu, \ \nu \sim iid(0, \sigma_\nu^2).$$

Note that this produces the following structure in the variance-covariance matrix:

$$\Omega_{esp1} = \sigma_\nu^2 (I_N + \rho_1 A)(I_N + \rho_1 A)^\top = \sigma_\nu^2 (I_N + 2\rho_1 A + \rho_1^2 A^2).$$

Although this would capture some of the features developed above, it imposes restrictions on the parameters, i.e., ρ and ρ_1^2 (see, for instance Kelejian and Prucha, 1999, 2010).

An alternative specification can be obtained from an spatial autoregressive model where spatial lags of the dependent variable (i.e., $A \cdot y$) are used as independent variable, in which case we have

$$\Omega_{esp2} = \sigma_\nu^2 (I_N - \rho_2 A)^{-1} (I_N - \rho_2 A)^{-1\top},$$

where ρ_2 is the first order spatial autoregressive parameter. Under some conditions this can be written in terms of A and its powers.

The added value of the paper can be seen in comparison to this spatial model. These models require a pre-specification of the adjacency matrix, which may be subject to misspecification. Once this is given there is a fixed relation among the variance-covariance components that multiply the powers of A, i.e., either ρ_1 or ρ_2. The potential gains from using a subgraph network error component model rather than a spatial model would depend on the flexibility of adding additional components in ε instead of a correlation structure of a single component ν.

3 ANOVA consistent variance components estimators

Here we consider simple consistent estimators of the variance components using ANOVA-type decompositions.

Consider the following statistics:

$$S_1 = \frac{1}{N} \sum_{i=1}^{N} u_i^2,$$

$$S_2 = \frac{1}{M} \sum_{i=1}^{N-1} \sum_{s>i}^{N} a_{is} u_i u_s,$$

$$S_3 = \frac{1}{T} \sum_{i=1}^{N-2} \sum_{s>i}^{N-1} \sum_{r>s}^{N} a_{is} a_{sr} a_{ir} u_i u_s.$$

S_1 contains the usual sum of squared errors. Note that for each vertex there will be at most $N-1$ edges to which it belongs and $N-2$ triangles. Moreover, each edge will be repeated twice for undirected graphs, one for each vertex, and each triangle will be repeated three times, one for each vertex. Then,

$$E[S_1 \mid x, G] = \sigma_\nu^2 + \sigma_\mu^2 \frac{2M}{N} + \sigma_\delta^2 \frac{3T}{N},$$

where $E[S_1 \mid x, G]$ is the (conditional) variance of a vertex.

S_2 contains the cross products of the error terms, which corresponds to the number of edges M. This corresponds to the existing active links (i.e., $a_{is} = 1$, $s > i$). For each active link, there could be at most $N-2$ triangles that can be formed from it. Now, each triangle will be repeated three times for each link. That is, for S_2 if we have an edge, say (i,s), that belongs to a triangle, say (i,s,r), such that $r > s > i$, then the triangle effect $\delta_{(i,s,r)}$ will appear in the edges (i,s), (s,r), and (i,s). Thus, each triangle will be found 3 times for every edge. Thus,

$$E[S_2 \mid x, G] = \sigma_\mu^2 + \sigma_\delta^2 \frac{3T}{M},$$

where $E[S_2 \mid x, G]$ is the (conditional) covariance of two vertices that have a common edge.

Finally, S_3 computes the cross products for active triangles (i.e., $a_{is} = a_{sr} = a_{ir} = 1$, $r > s > i$). Note that for S_3 if we have a triangle, say (i,s,r), then two nodes, say i and s, must share both $\mu_{(i,s)}$ and $\delta_{(i,s,r)}$. Then,

$$E[S_3 \mid x, G] = \sigma_\mu^2 + \sigma_\delta^2,$$

where $E[S_3 \mid x, G]$ is the (conditional) covariance of two vertices that have common edge and triangle(s).

In the absence of triangle effects, i.e., $\sigma_\delta^2 = 0$, the model simplifies to

$$\sigma_\nu^2 = E[S_1 \mid x, G] - E[S_2 \mid x, G]\frac{2M}{N},$$
$$\sigma_\mu^2 = E[S_2 \mid x, G],$$

such that the non-negativity restrictions are $E[S_2 \mid x, G] \geq 0$ and $\frac{E[S_1|x,G]}{E[S_2|x,G]} \geq \frac{2M}{N}$, such that the ratio of the variance of a vertex to the covariance of two random vertices needs to be bigger than the average number of edges per vertex. First, take for instance a cycle graph, a 2-regular graph with all vertices of degree 2 such that $M = N$. For this case the variance of the vertices need to be at least twice the covariance. Second, consider a complete graph with $M = N(N-1)/2$. In this case, the ratio of variance to covariance needs to grow faster than the number of vertices.

In the absence of edge effects, $\sigma_\mu^2 = 0$, the model simplifies to

$$\sigma_\nu^2 = E[S_1 \mid x, G] - E[S_3 \mid x, G]\frac{3T}{N},$$
$$\sigma_\delta^2 = E[S_3 \mid x, G],$$

such that the non-negativity restrictions are $E[S_3 \mid x, G] \geq 0$ and $\frac{E[S_1|x,G]}{E[S_3|x,G]} \geq \frac{3T}{N}$, such that the ratio of the variance of a vertex to the covariance of two random vertices needs to be bigger than the average number of triangles per vertex.

For the edge and triangle effects model, solving for $(\sigma_\nu^2, \sigma_\mu^2, \sigma_\delta^2)$ gets

$$\sigma_\nu^2 = E[S_1 - \frac{E[S_2 \mid x, G] - E[S_3 \mid x, G]\frac{3T}{M}}{1 - \frac{3T}{M}}\frac{2M}{N} - \frac{E[S_3] - E[S_2]}{1 - \frac{3T}{M}}\frac{3T}{N},$$
$$\sigma_\mu^2 = \frac{E[S_2 \mid x, G] - E[S_3 \mid x, G]\frac{3T}{M}}{1 - \frac{3T}{M}},$$
$$\sigma_\delta^2 = \frac{E[S_3 \mid x, G] - E[S_2 \mid x, G]}{1 - \frac{3T}{M}}.$$

For this case the non-negativity restrictions imply: (i) $\frac{E[S_3|x,G]}{E[S_2|x,G]} \geq 1$, (ii) $\frac{E[S_2|x,G]}{E[S_3|x,G]} \geq \frac{3T}{M}$, (iii) $\frac{E[S_1|x,G]}{E[S_2|x,G]} \geq \frac{2M}{N}$, and (iv) $\frac{E[S_1]}{E[S_3]} \geq \frac{3T}{MN}$. Restriction (i) implies that the covariance among vertices that belong to a triangle must be larger than the covariance of vertices that share a link. Restriction (ii) states that the ratio $\frac{E[S_3|x,G]}{E[S_2|x,G]}$ cannot exceed the average number of triangles per edge. Restriction (iii) correspond to the number average number of links per vertex. Restriction (iv) is a combination of the above with no clear interpretation.

The consistent estimators are then constructed by defining $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_3$, where the OLS residuals $\hat{\epsilon}$ are used, and the non-negativity constraints are imposed.

4 Specification tests for undirected unweighted graphs

The log likelihood function for this problem is given by

$$\mathrm{L}(\beta,\theta) \;\propto\; -\frac{1}{2}\ln|\Omega| - \frac{1}{2}\varepsilon^{\top}\Omega^{-1}\varepsilon,$$

with $\theta = (\sigma_{\nu}^2, \sigma_{\mu}^2, \sigma_{\delta}^2)$, $\varepsilon = y - x\beta$, and Ω is given by equation (5). In this model we have that the Fisher information matrix is block diagonal in terms of β and θ. This feature also applies to non-Gaussian error components, where in fact OLS estimators for β are consistent. In turn, this simplifies the subsequent algebra where we only consider θ for constructing our LM tests.

Let $\theta \in \Theta \subseteq \mathbb{R}^p$, where p is the dimension of θ. Using the formulas in Harville (1977, p.236) the score functions can be expressed as

$$s_r(\theta) = \partial L/\partial\theta_r = -\frac{1}{2}\mathrm{tr}(\Omega^{-1}\partial\Omega/\partial\theta_r) + \frac{1}{2}\{\epsilon^{\top}\Omega^{-1}(\partial\Omega/\partial\theta_r)\Omega^{-1}\epsilon\},$$

for a parameter θ_r in θ, $1 \leq r \leq p$. The information matrix $\mathcal{J}$ can be obtained for for $1 \leq r, k \leq p$. as

$$\begin{aligned}\partial^2 L/\partial\theta_r\partial\theta_k \;&=\; \frac{1}{2}\mathrm{tr}\left(\Omega^{-1}\left\{\frac{\partial^2\Omega}{\partial\theta_r\partial\theta_k} - \frac{\partial\Omega}{\partial\theta_r}\Omega^{-1}\frac{\partial\Omega}{\partial\theta_k}\right\}\right)\\ &\quad +\frac{1}{2}\epsilon^{\top}\Omega^{-1}\left(\frac{\partial\Omega}{\partial\theta_r\partial\theta_k} - 2\frac{\partial\Omega}{\partial\theta_r}\Omega^{-1}\frac{\partial\Omega}{\partial\theta_r}\right)\Omega^{-1}\epsilon,\end{aligned}$$

and

$$\mathcal{J}_{rk}(\theta) \equiv -\mathrm{E}(\partial^2 L/\partial\theta_r\partial\theta_k) = \frac{1}{2}tr\left(\Omega^{-1}\frac{\partial\Omega}{\partial\theta_r}\Omega^{-1}\frac{\partial\Omega}{\partial\theta_k}\right).$$

Note that

$$\partial\Omega/\partial\sigma_\nu^2 = I_N, \tag{7}$$

$$\partial\Omega/\partial\sigma_\mu^2 = BB^\top, \tag{8}$$

$$\partial\Omega/\partial\sigma_\delta^2 = CC^\top. \tag{9}$$

In order to construct LM tests, first note that the block diagonality between β and θ allow us to focus on the scores corresponding to θ only. Second, consistent estimators of θ under the null can be obtained using an ANOVA-type analysis as in Section 3. Hence our tests will be based on Neyman's $C(\alpha)$ principle, which produces tests that are asymptotically equivalent to likelihood based LM tests under $\sqrt{N}$-consistent non-maximum likelihood estimation of the nuisance parameters. See Bera and Bilias (2001) for a discussion.

Consider a partition of $\theta = (\theta_1^\top, \theta_2^\top)^\top$, where θ_2 contains the parameters under the corresponding null hypothesis $H_0^2 : \theta_2 = 0$, and θ_1 the nuisance parameters that need to be estimated. In our particular case, θ will be partitioned into either $\theta_1 = \sigma_\nu^2, \theta_2 = \sigma_\mu^2$ when we want to test for the presence of edge network effects assuming $\sigma_\delta^2 = 0$, $\theta_1 = \sigma_\nu^2, \theta_2 = \sigma_\delta^2$ when we want to test for the presence of edge and triangle network effects assuming $\sigma_\mu^2 = 0$, $\theta_1 = \sigma_\nu^2, \theta_2 = (\sigma_\mu^2, \sigma_\delta^2)$ when we want to test for the presence jointly of edge and triangle network effects, $\theta_1 = (\sigma_\nu^2, \sigma_\mu^2), \theta_2 = \sigma_\delta^2$ when we want to test for the presence pf triangle effects assuming edge effects or $\theta_1 = (\sigma_\nu^2, \sigma_\delta^2), \theta_2 = \sigma_\mu^2$ when we want to test for the presence pf triangle effects assuming edge effects. Correspondingly, the score will be partitioned as $s(\theta) = (s_1(\theta)^\top, s_2(\theta)^\top)^\top$, and the information matrix as $\mathcal{J}(\theta) = \begin{pmatrix} \mathcal{J}_{11}(\theta) & \mathcal{J}_{12}(\theta) \\ \mathcal{J}_{21}(\theta) & \mathcal{J}_{22}(\theta) \end{pmatrix}$.

Conditional LM statistics for H_0^2 under maximum likelihood estimation are defined as

$$LM_2(\theta) = s_2(\theta)^\top \{\mathcal{J}_{22}(\theta) - \mathcal{J}_{21}(\theta)\mathcal{J}_{11}^{-1}(\theta)\mathcal{J}_{12}(\theta)\}^{-1} s_2(\theta).$$

Neyman's C(α) adjusted scores are defined as

$$s_{2\cdot 1}(\theta) \equiv s_2(\theta) - \mathcal{J}_{21}(\theta)\mathcal{J}_{11}^{-1}(\theta)\mathcal{J}_{12}(\theta)s_1(\theta).$$

Then, the Neyman's C(α) LM statistic is

$$LM_{2\cdot 1}(\theta) = s_{2\cdot 1}(\theta)^{\top}\{\mathcal{J}_{22}(\theta) - \mathcal{J}_{21}(\theta)\mathcal{J}_{11}^{-1}(\theta)\mathcal{J}_{12}(\theta)\}^{-1}s_{2\cdot 1}(\theta).$$

A well known result is that $LM_{2\cdot 1}(\hat{\theta}) \xrightarrow{d} \chi^2_{dim(\theta_2)}$, where $\hat{\theta}$ is a $\sqrt{N}$-consistent estimator under the corresponding null hypothesis. Note that when we estimate the parameters under the joint null $\sigma^2_\mu = \sigma^2_\delta = 0$, the ML estimators of β and σ^2_ν coincide with the least-squares estimators.

Consider now Bera and Yoon (1993) locally size-robust type statistics (BY test hereafter). For this, consider a new partition of $\theta = (\theta_1, \theta_2, \theta_3)' = (\theta_1, \theta_{23})'$ where we want to test for the null hypothesis H_0^2, we consider θ_1 as nuisance parameters to be estimated, but the validity of the test is affected by the validity of $H_0^3 : \theta_3 = 0$. Global valid tests for H_0^2 would require consistent estimators of θ_3 as in the construction of the conditional LM statistics above. In practice, however, estimators of θ_3 may be cumbersome or it might suffer identification conditions under the null. Thus, Bera and Yoon (1993) has been successfully implemented to test one particular null without estimating the other nuisance parameter θ_3. This procedure is valid under $\sqrt{N}$-*local* deviations of H_0^3, but different empirical studies confirmed its validity for non-local deviations too. In our particular case, the parameter will be partitioned as $\theta_1 = \sigma^2_\nu, \theta_2 = \sigma^2_\mu, \theta_3 = \sigma^2_\delta$. This procedure thus allows us to test for triangle effects but without estimating edge effects variance component, even when we are estimating under the joint null hypothesis $H_0^2 \& H_0^3 : \sigma^2_\mu = \sigma^2_\delta = 0$, which is just least-squares estimation. The statistic is constructed as in Bera, Montes-Rojas, and Sosa-Escudero (2010, 2017) for non-maximum likelihood estimation as

$$LM_{2(3)\cdot 1}(\theta) = s_{2(3)\cdot 1}(\theta)'[\mathcal{J}_{2(3)\cdot 1}(\theta)]^{-1}s_{2(3)\cdot 1}(\theta),$$

where

$$\begin{aligned} s_{2(3)\cdot 1}(\theta) &= s_{2\cdot 1}(\theta) - \mathcal{J}_{23\cdot 1}(\theta)\mathcal{J}_{33\cdot 1}^{-1}(\theta)s_{3\cdot 1}(\theta), \\ \mathcal{J}_{2(3)\cdot 1}(\theta) &= \mathcal{J}_{22\cdot 1}(\theta) - \mathcal{J}_{23\cdot 1}(\theta)\mathcal{J}_{33\cdot 1}^{-1}(\theta)\mathcal{J}_{32\cdot 1}(\theta), \\ \mathcal{J}_{22\cdot 1}(\theta) &= \mathcal{J}_{22}(\theta) - \mathcal{J}_{21}(\theta)\mathcal{J}_{11}^{-1}(\theta)\mathcal{J}_{12}(\theta), \\ \mathcal{J}_{33\cdot 1}(\theta) &= \mathcal{J}_{33}(\theta) - \mathcal{J}_{31}(\theta)\mathcal{J}_{11}^{-1}(\theta)\mathcal{J}_{13}(\theta), \\ \mathcal{J}_{23\cdot 1}(\theta) &= \mathcal{J}_{23}(\theta) - \mathcal{J}_{23,1}(\theta)\mathcal{J}_{11}^{-1}(\theta)\mathcal{J}_{1,23}(\theta). \end{aligned}$$

Then $LM_{2(3)\cdot 1}(\hat{\theta}) \xrightarrow{d} \chi^2_{dim(\theta_2)}$ for $\hat{\theta}$ being a consistent estimator under the joint null hypothesis $H_0^2 \& H_0^3 : \sigma_\mu^2 = \sigma_\delta^2 = 0$ and for $\theta_3 = o(1/\sqrt{N})$.

In sum, the LM tests considered are:

- LM_μ: LM test for $H_0 : \sigma_\mu^2 = 0$ when σ_ν^2 is estimated as mean squared error (MSE) after OLS estimation and $\sigma_\delta^2 = 0$ is assumed.
- LM_δ: LM test for $H_0 : \sigma_\delta^2 = 0$ when σ_ν^2 is estimated as MSE after OLS estimation and $\sigma_\mu^2 = 0$ is assumed.
- $LM_{\mu,\delta}$: LM test for $H_0 : \sigma_\mu^2 = \sigma_\delta^2 = 0$ when σ_ν^2 is estimated as MSE after OLS estimation.
- $LM_{\mu(\delta)}$: BY test for $H_0 : \sigma_\mu^2 = 0$ when σ_ν^2 is estimated as MSE after OLS estimation and $\sigma_\delta^2 = 0$ is allowed to have local deviations.
- $LM_{\delta(\mu)}$: BY test for $H_0 : \sigma_\delta^2 = 0$ when σ_ν^2 is estimated as MSE after OLS estimation and $\sigma_\mu^2 = 0$ is allowed to have local deviations.
- $LM_{\delta-\mu}$: LM test for $H_0 : \sigma_\delta^2 = 0$ when $(\sigma_\nu^2, \sigma_\mu^2)$ is estimated as in Section 3 after OLS estimation.

5 Monte Carlo experiments

This section explores the small sample performance of the proposed tests through a Monte Carlo experiment. We will consider the following simple regression model:

$$y_i = x_i\beta + \varepsilon_i,$$
$$\varepsilon_i = \nu_i + \sum_{i=1}^{N}\sum_{s>i}^{N} a_{is}\mu_{(is)} + \sum_{i=1}^{N}\sum_{s>i}^{N}\sum_{r>s}^{N} a_{is}a_{sr}a_{ir}\delta_{(isr)},$$
$$i = 1, 2, ..., N,$$

where $A = \{a_{ir}\}$ is an adjacent contiguity matrix. We assume $x_i \sim \ iid\ N(0,1)$, $\beta = 1$, $\nu_i \sim \ iid\ N(0,10)$, $\mu_{(is)} \sim \ iid\ N(0,\sigma_\mu^2)$ and $\delta_{(isr)} \sim \ iid\ N(0,\sigma_\delta^2)$.

We consider $N \in \{100, 225, 400\}$ and simulate two types of networks. First, we consider an Erdös-Rényi random graph where links are randomly generated with a given probability p_N, i.e., $Prob(a_{ir} = 1) = p_N$, $i, r = 1, \ldots, N, i \neq r$. For the Erdös-Rényi graphs we have on average a constant proportion of vertices and edges, N/M, using $p_{100} = 0.05, p_{225} = 0.05 \times 100/225, p_{400} = 0.05 \times 100/400$. In this case, the number of triangles per node is also constant on average. Second, a queen-type spatial structure where edges are generated according to queen contiguity, i.e., for a squared board with number of rows and columns $n = \sqrt{N}$, for $i = 1, \ldots, N$, $a_{ir} = 1$ if $r \in \{i-1, i+1, i-n-1, i-n, i-n+1, i+n-1, i+n, i+n+1\}$ with $1 \leq r \leq N$, and $a_{ir} = 0$ otherwise. Note that the considered spatial-type model has a similar number of triangles and edges for each node, i.e., 8 edges and triangles for a node that is not on the border of the board.

First, we consider the empirical size results where we impose the absence of both edge and triangle random effects, $\sigma_\mu^2 = \sigma_\delta^2 = 0$ in Table 7. In all cases, marginal, joint and robust tests have the appropriate size, for all levels of significance.

Second, we consider the empirical power and size-robustness for $(\sigma_\mu^2, \sigma_\delta^2) \in \{0, 1, \ldots, 10\}^2$ in figures 1 and 4. In each case the (a) figure report the rejection rates as we increase the value of σ_μ^2, and the (b) the rejection rates for different values of σ_μ^2.

Figures 1 and 2 report the tests for detecting edge heterogeneity, that is, $\sigma_\mu^2 > 0$. Note that the marginal tests LM_μ has the largest power performance for changes in σ_μ^2 (see top figures (a)), followed by the joint tests $LM_{\mu\delta}$. However, the marginal test also rejects in the direction of $\sigma_\delta^2 > 0$, as the bottom (b) figures show, that is, it is not robust to the presence of triangle effects. The BY tests are constructed to be able to detect departures from $\sigma_\mu^2 = 0$, while being robust to the presence of $\sigma_\delta^2 > 0$, without estimating σ_δ^2. The BY robust test have good power performance in figures 1-(a), in fact, close to the joint test, but it has low power in the Queen spatial more complex network model, as shown in figure 2-(a). Nevertheless, the BY test is robust to deviations in $\sigma_\delta^2 > 0$, as seen in figures 1-(b) and 2-(b).

Tests for triangle effects have a similar performance to those of edge effects. As in the previous paragraph, the tests have the expected rejection rates in the direction of $\sigma_\delta^2 > 0$, and the BY robust test have correct size for $\sigma_\mu > 0$. Note that the conditional test $LM_{\delta-\mu}$ estimates σ_μ^2, and as such it should be robust to misspecification in edge effects. For this case the BY robust tests outperforms it in terms of size and power in the Erdös-Rényi random graph model, and it is very close to the conditional tests in the Queen spatial structure.

6 Empirical application

Network analysis of the degree of interconnectedness in the financial system can inform policymakers on optimal bank resolutions mechanisms and how regulation can help to reduce instability. Empirical networks have been used for stress test exercises (see Upper, 2011, for a comprehensive review). Network centrality measures, developed to assess centrality in other contexts and adapted to the context of financial networks, can guide national authorities in their assessment of the systemic importance of financial and non-financial institutions. In the financial economic literature network analysis has mostly been applied to payment systems, interbank lending markets, and more recently extended to capture the mutual exposure of financial institutions to other asset classes, including derivatives contracts, in a multilayer networks framework (Langfield, Liu, and Ota, 2014; Bargigli, di Iasio, Infante, Lillo, and Pierobon, 2015; Molina-Borboa, Martinez-Jaramillo, Lopez-Gallo, and van der Leij, 2015; Poledna, Molina-Borboa, Martínez-Jaramillo, van der Leij, and Thurner, 2015).

Network positioning could affect banks' interest rates by different mechanisms. First, in line with Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), dense interconnections serve as a mechanism for the propagation of shocks, leading to a more fragile financial system. As such, banks that are more connected may be perceived by the market as fragile. The same banks can be perceived as 'too-interconnected-to-fail' such that rather than fragile, those banks are perceived as more likely to be bailout (see for instance Battiston, Puliga, Kaushik, Tasca, and Caldarelli, 2012). This is similar to the too-big-to-fail effect observed in other interbank markets. Second, as argued by Booth, Gurun, and Zhang (2014), financial institutions with more extensive and strategic financial networks, can more efficiently acquire and process information due to their better access to order flows (see, e.g., Temizsoy, Iori, and Montes-Rojas, 2015). Third, banks with higher centrality within the network have better access to liquidity and are able to charge larger intermediation spreads. Previous empirical evidence (Angelini, Nobili, and Picillo, 2011; Bech, Chapman, and Garratt, 2010; Temizsoy, Iori, and Montes-Rojas, 2017) suggests that being systemically more important, in term of size or connectedness, can explain part of the cross-sectional variation in banks' borrowing costs before and during the global financial crisis.

The centrality indicators used in the empirical analysis are constructed from measures of distance of a bank from the other banks in the network, where distance is expressed in terms of paths of length one, i.e. the number of incoming or outgoing links. Below we consider two basic measures of centrality. A local measure given by the total degree and a global measure given by eigenvalue centrality.

We apply this analysis to the interbank overnight Call market in Argentina. In this market, banks talk to each other to lend or borrow most of the needed funds to satisfy Central Bank regulation requirements at the end of the day. While the participating banking market is small (aprox. 50 banks), Argentina is known for recurrent financial collapses, partial and full defaults on different contracts, all of which affects the financial sector.

We use daily data for the period 1st January 2015 to 31st December 2018. While the Call market has contracts from 1 up to 7 days long, we keep the sample of transactions on 1 to 4 days only, thus having a sample of overnight and weekends only. Networks are constructed on a monthly basis to capture the rich strategic relationships that can be formed. Thus we consider 48 cross-section network structures, corresponding to one for each month.

Consider a network G_t of N_t banks, where t indexes time (i.e. months). The dependent variable of interest is π_{it} that is the net profit obtained in a given month from all lending and borrowing transactions, and defined as

$$\pi_{it} = \sum_{h=1}^{H_t} \sum_{j=1, j\neq i}^{N_t} \left(L_{ij,h} r_{ij,h} - B_{ji,h} r_{ji,h} \right),$$

where $h = 1, 2, ..., H_t$ are the days within month t, N_t corresponds to the banks that participate in the Call market during month t, $L_{ij,h}$ is the total amount i lent to j on day h at rate $r_{ij,h}$, $B_{ij,h}$ is the total amount i borrow from j on day h at rate $r_{ji,h}$.

Following Temizsoy, Iori, and Montes-Rojas (2015, 2017) we consider a regression model of the form

$$\pi_{it} = \beta_0 + \beta_1 V_{it} + \beta_2 Liq_{it} + \beta_3 Degree_{it} + \beta_3 Eigen_{it} + \epsilon_{it}, \qquad (10)$$

for each month t, and for all banks i that participate in that month t. V_{it} is the log of the total volume of lending and borrowing of bank i during month t. Liq_{it} is a liquidity index defined as in Afonso and Lagos (2015), $\frac{L_{i,t}+B_{i,t}-|L_{i,t}-B_{i,t}|}{L_{i,t}+B_{i,t}}$, where $L_{i,t}$ and $B_{i,t}$ are the total amount lent and borrowed, respectively, by bank i during month t. $Degree_{i,t}$ is the total degree (both in and out) and $Eigen_{it}$ is the undirected eigenvalue centrality of bank i in the network of month t.

We consider different regression models one for each month from a total of 48 months (4 years). In each case we evaluate the proposed tests, $LM_{\mu,\delta}$, LM_{μ}, LM_{δ}, $LM_{\mu(\delta)}$ and $LM_{\delta(\mu)}$, and we compare them with Anselin, Bera, Florax, and Yoon (1996) canonical tests for spatial dependence. In particular, one for spatial error autocorrelation, and one for spatial lag autocorrelation of the dependent variable.

Several features of the exercise can be highlighted.

First, the proposed subgraph tests are different from test for spatial dependence. In order to show this we compute the joint test results for edge&triangle effects, $LM_{\mu,\delta}$, and those of Anselin, Bera, Florax, and Yoon (1996) for spatial error and spatial lag specifications. Figure 5 shows the p-values (in log-scale) of the tests for each month. The analysis reveals that in most cases, when $LM_{\mu,\delta}$ rejects (i.e. when the tests detect the presence of subgraph network random effects, of any type), spatial tests do not; and when spatial tests reject, only a few tests reject. This indicates that our proposed subgraph network structure is different from what would be captured by an model of spatial autocorrelation (in the error structure or lagged dependent variable). The same pattern appears if we consider Anselin, Bera, Florax, and Yoon (1996) robust LM tests[1] also compared with the results of

[1] These correspond to Bera and Yoon (1993) type LM tests were each type of spatial autocorrelation is made robust to the presence of the other type.

$LM_{\mu,\delta}$ (see Figure Figure 6).

Second, there is heterogeneity across months in terms of the presence of edge, triangle, and edge&triangle effects. Figure 7 plots the test results for LM_μ (marginal LM test for edge effects) and LM_δ (marginal LM test for triangle effects). The figure reveals that there are more cases of only edge effects, while less so with triangle effects detected. 9 cases out of 48 show that both effects are present at 10% significance level and 6 out of 48 at 5% significance level. When we explore this further using the robust LM tests in Figure 8, $LM_{\mu(\delta)}$ for edge effects controlling for triangle effects, and $LM_{\delta(\mu)}$ for triangle effects controlling for edge effects, only 6 cases out of 48 have both effects at 10% significance level and 3 out of 48 at the 5% significance level.

7 Conclusion

This paper develops a simple model of subgraph network random effects that can be used to estimate the variance-covariance matrix in a linear OLS set up with network data. It focuses on evaluating the appropriate level of effects, using the example of links' and triangles' effects as random components, and constructung s battery of specification tests.

Monte Carlo evidence shows that the tests correctly identify the type of network structure. These varies depending on the network structure.

Finally, an empirical application to the interbank market in Argentina reveals differences when compared with the two canonical types of spatial autocorrelation in Anselin, Bera, Florax, and Yoon (1996). That is, the proposed subgraph network error component captures different features of the data than those of spatial models. The application also shows heterogeneity across periods.

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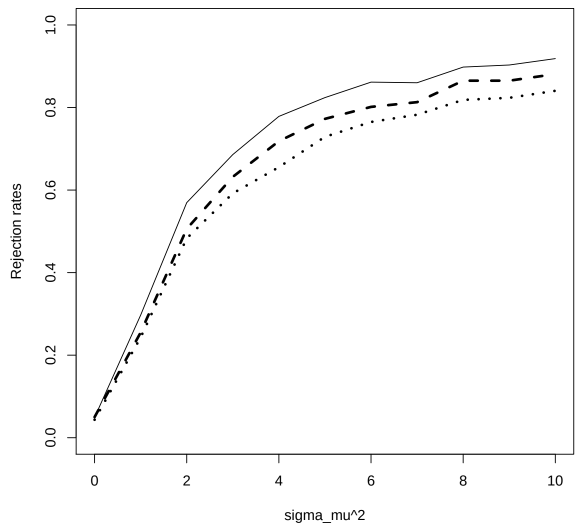
Table 1: Empirical size

N	LM_{μ}	LM_{δ}	$LM_{\mu,\delta}$	$LM_{\mu(\delta)}$	$LM_{\delta(\mu)}$	$LM_{\delta-\mu}$
Erdös-Rényi random graph						
Size 1%						
100	0.009	0.016	0.0145	0.009	0.0165	0.0115
225	0.012	0.0115	0.015	0.013	0.012	0.009
400	0.013	0.012	0.0085	0.0095	0.0075	0.007
Size 5%						
100	0.043	0.05	0.0465	0.042	0.052	0.041
225	0.052	0.0485	0.0495	0.052	0.0495	0.041
400	0.047	0.0475	0.049	0.046	0.046	0.0435
Size 10%						
100	0.082	0.0885	0.0855	0.089	0.092	0.0765
225	0.1045	0.092	0.102	0.098	0.0995	0.0875
400	0.089	0.087	0.093	0.0965	0.099	0.0915
Spatial queen structure						
Size 1%						
100	0.0115	0.0105	0.0105	0.01	0.011	0.0115
225	0.0075	0.0065	0.012	0.0145	0.0135	0.014
400	0.0085	0.0085	0.0095	0.012	0.011	0.011
Size 5%						
100	0.0475	0.0515	0.047	0.048	0.044	0.046
225	0.045	0.039	0.0565	0.0595	0.052	0.0525
400	0.046	0.0465	0.049	0.0535	0.049	0.0505
Size 10%						
100	0.0965	0.0975	0.0955	0.094	0.09	0.097
225	0.0965	0.09	0.1	0.1085	0.1115	0.1125
400	0.0935	0.0965	0.098	0.096	0.0995	0.1015

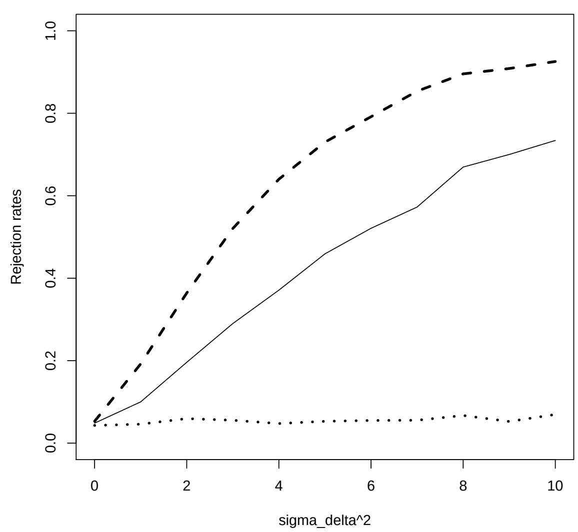
Notes: Monte carlo experiments based on 2000 replications.

Figure 1: LM tests for edge effects, $\sigma_\mu^2 = 0$, Erdös-Rényi random graph

(a)



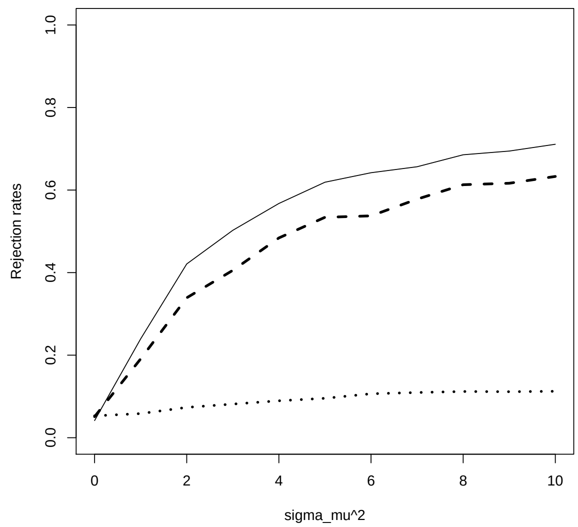
(b)



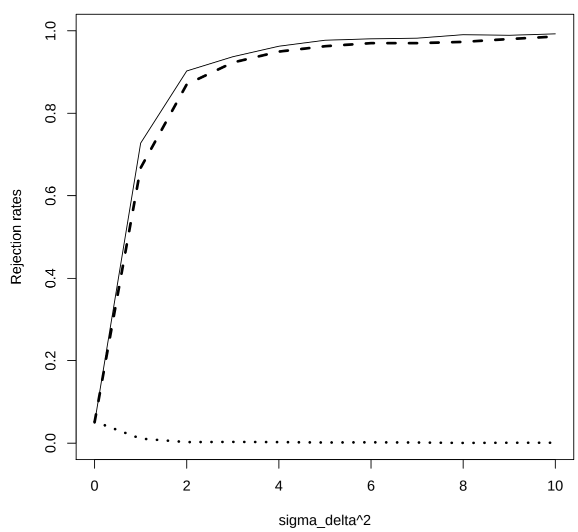
Notes: Monte carlo experiments based on 2000 replications. Solid line: LM_μ. Dashed line: $LM_{\mu\delta}$. Dotted line: $LM_{\mu(\delta)}$.

Figure 2: LM tests for edge effects, $\sigma_\mu^2 = 0$, Queen spatial structure

(a)



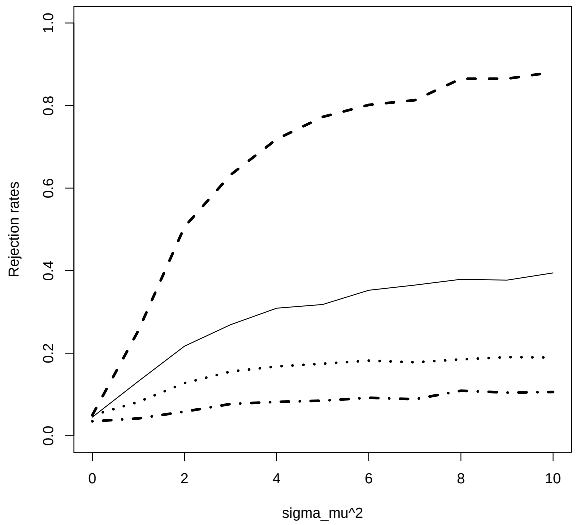
(b)



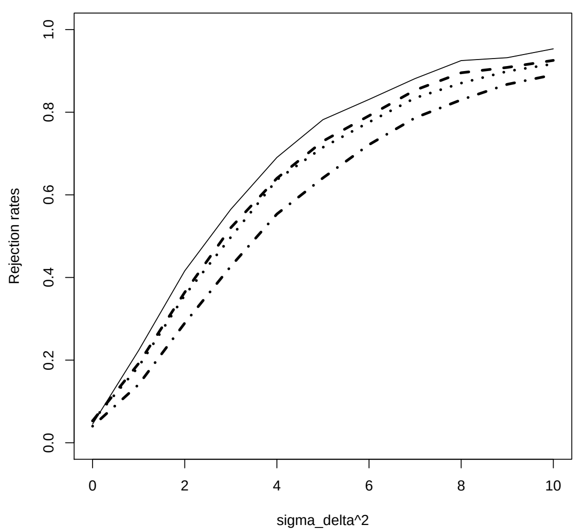
Notes: Monte carlo experiments based on 2000 replications. Solid line: LM_μ. Dashed line: $LM_{\mu\delta}$. Dotted line: $LM_{\mu(\delta)}$.

Figure 3: LM tests for triangle effects, $\sigma_\delta^2 = 0$, Erdös-Rényi random graph

(a)



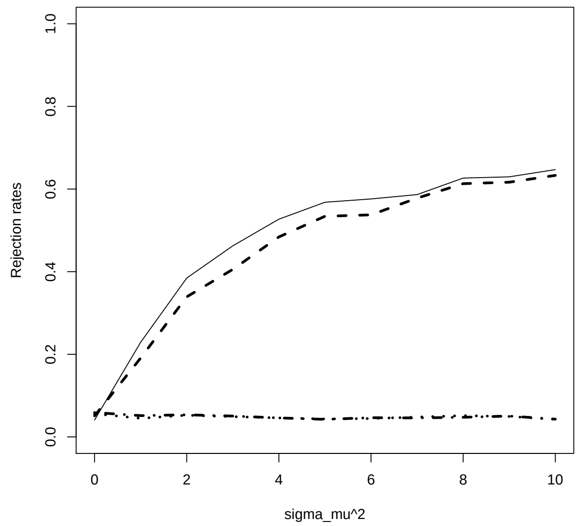
(b)



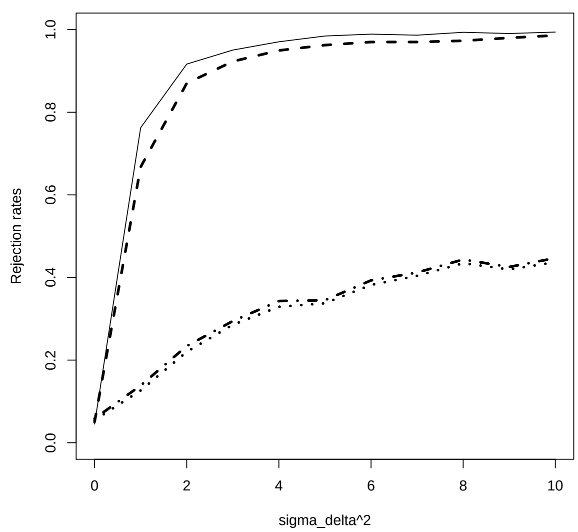
Notes: Monte carlo experiments based on 2000 replications. Solid line: LM_δ. Dashed line: $LM_{\mu\delta}$. Dotted line: $LM_{\delta(\mu)}$. Dash-dot line: $LM_{\delta-\mu}$.

Figure 4: LM tests for triangle effcts, $\sigma_\delta^2 = 0$, Queen spatial structure

(a)

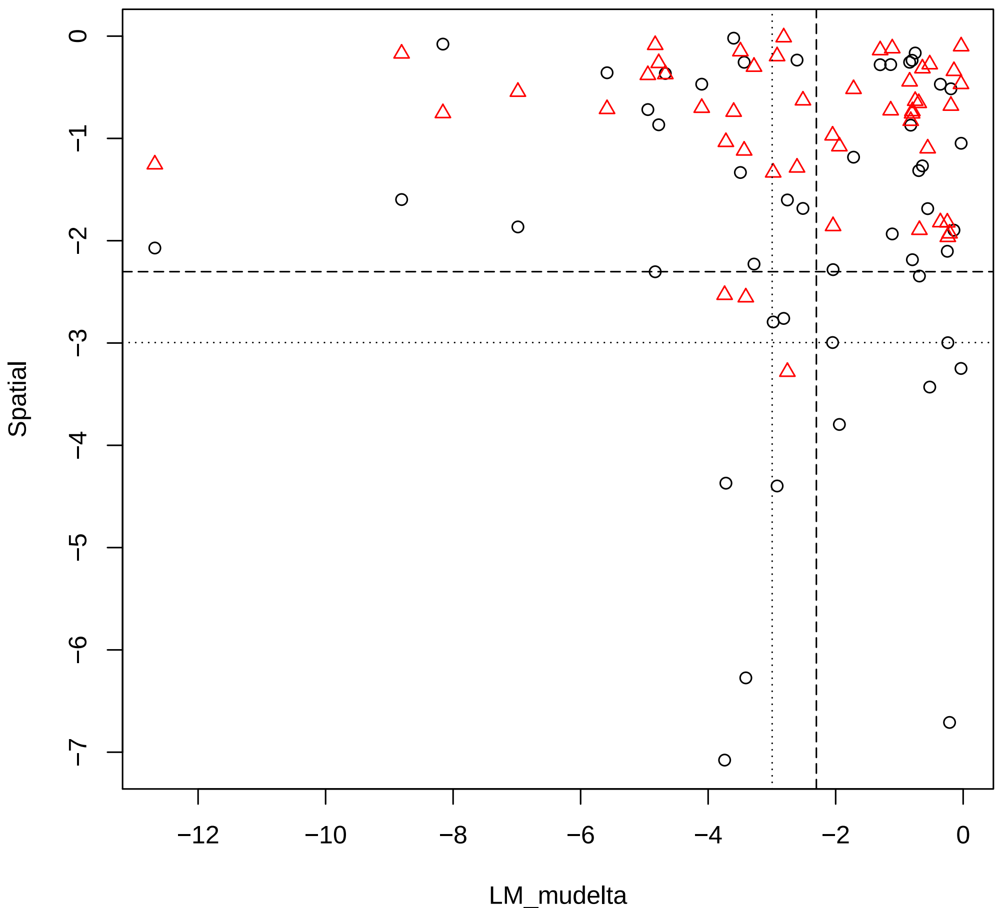


(b)



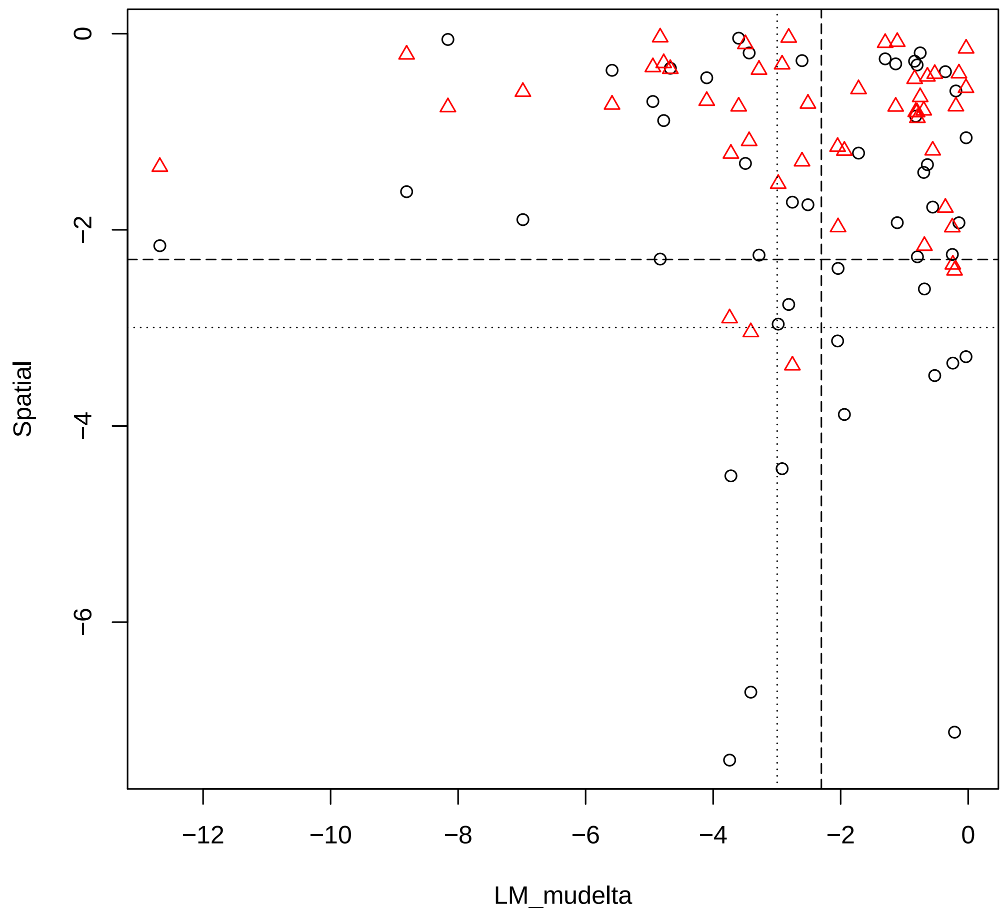
Notes: Monte carlo experiments based on 2000 replications. Solid line: LM_δ. Dashed line: $LM_{\mu\delta}$. Dotted line: $LM_{\delta(\mu)}$. Dash-dot line: $LM_{\delta-\mu}$.

Figure 5: Subgraph joint tests for edge and triangle effects and spatial LM test



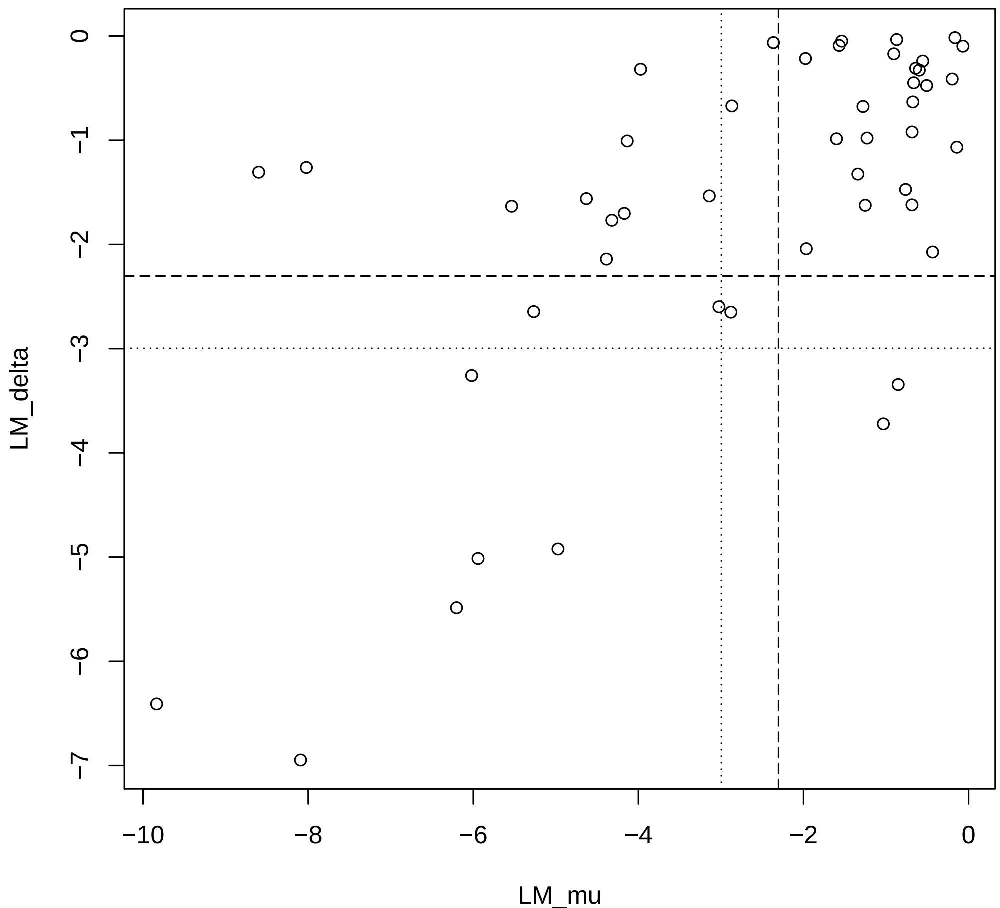
Note: P-values in log-scale. Dashed line is the 10% critical value and dotted line to the 5% critical values. Horizontal axis corresponds to joint test for edge and triangle effects ($LM_{\mu,\delta}$). Vertical axis corresponds to Anselin, Bera, Florax, and Yoon (1996) LM tests for spatial error (circles) and spatial lag (triangles).

Figure 6: Subgraph joint tests for edge and triangle effects and robust spatial LM test



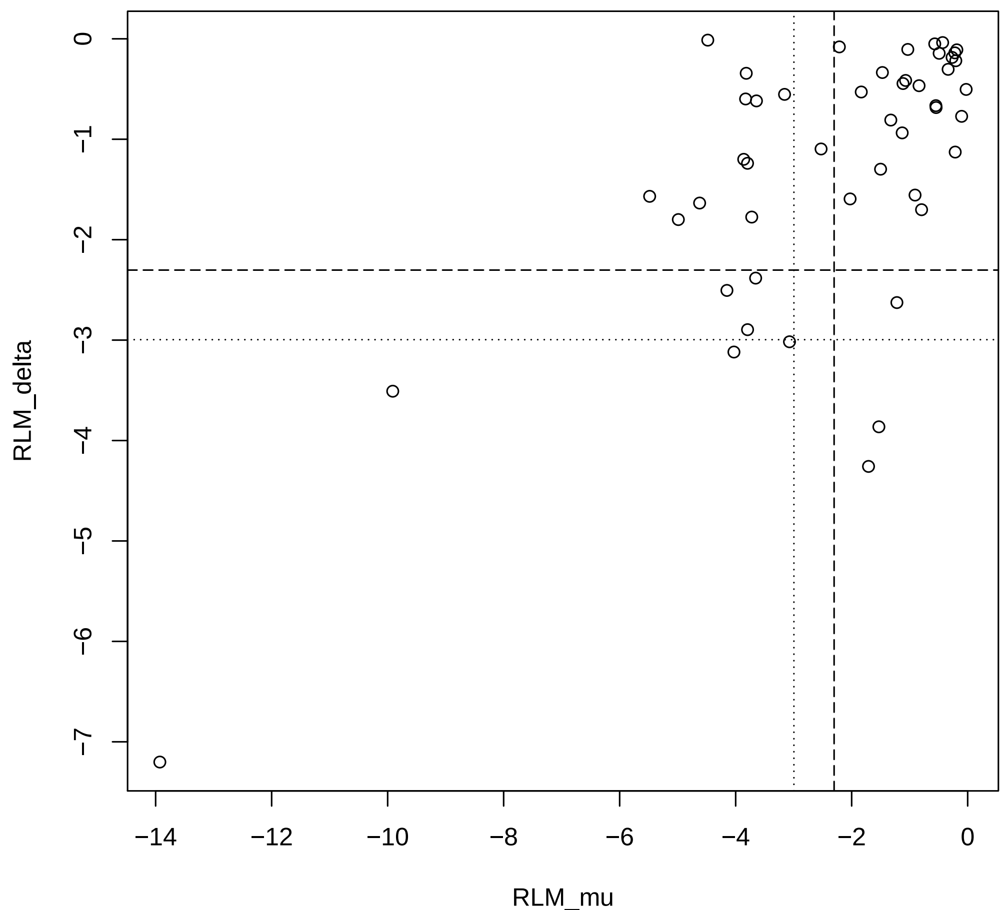
Note: P-values in log-scale. Dashed line is the 10% critical value and dotted line to the 5% critical values. Horizontal axis corresponds to joint test for edge and triangle effects ($LM_{\mu,\delta}$). Vertical axis corresponds to Anselin, Bera, Florax, and Yoon (1996) robust LM tests for spatial error (circles) and spatial lag (triangles).

Figure 7: Subgraph tests for edge and triangle effects



Note: P-values in log-scale. Dashed line is the 10% critical value and dotted line to the 5% critical values. Horizontal axis corresponds to tests for edge effects (LM_{μ}). Vertical axis corresponds to tests for triangle effects (LM_{δ}).

Figure 8: Robust subgraph tests for edge and triangle effects



Note: P-values in log-scale. Dashed line is the 10% critical value and dotted line to the 5% critical values. Horizontal axis corresponds to tests for edge effects robust to triangle effects ($LM_{\mu(\delta)}$). Vertical axis corresponds to tests for triangle effects robust to edge effects ($LM_{\delta(\mu)}$).