

Political Uncertainty and the Peso Problem*

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Abstract

This paper analyses the relation between political uncertainty and the Peso Problem in emerging markets. Initially, it is assumed that the country has a hard peg system (the present government will never devalue). As for the political opposition, however, it is open to the possibility of leaving the fixed regime when it comes to power. Assuming that the change of government follows a Poisson distribution, our model shows that the expectations of a devaluation under the subsequent new government may drive up country risk premium under the first government. Sovereign spreads in Argentina in 2001 are used to illustrate the argument.

Resumen

Este trabajo analiza la relación entre la incertidumbre política y el *Peso Problem* en mercados emergentes. Inicialmente, se asume que el país tiene un sistema de tipo de cambio fijo y que el gobierno actual nunca devaluará. Por otro lado, la oposición política está abierta a la posibilidad de dejar el régimen fijo si es que toma el poder en el futuro. Si se asume que el cambio del gobierno sigue una distribución de Poisson, nuestro modelo demuestra que las expectativas de una devaluación futura bajo el nuevo gobierno pueden inducir un incremento en el riesgo país bajo el primer gobierno. La situación de la Argentina en 2001 se utiliza para ilustrar el argumento.

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1. Introduction

In emerging market countries, political uncertainty is not uncommon: in particular, the transfer of political power is not always subject to normal terms of election. In these circumstances, market expectations must not only take into account the policy of the present government, but also incorporate future decisions of its potential successor. This could increase country risk even when the first government is fully committed to a pegged exchange rate, particularly if the succeeding government is known to be considering devaluation and strategic default.

This paper develops a model suitable for situations of political uncertainty and substantial dollarisation -both pervasive factors in emerging markets. The former has been studied by Alesina *et al.* (1996), who define “political instability as the propensity of a government collapse”. Dollarisation of sovereign debt has been at the centre of the debate on the *original sin* (Eichengreen & Hausmann 1999).

In the present model it is assumed that the country under analysis has two possible governments with different policy preferences: the existing government is fully committed to maintaining the peg, and the succeeding government is not. Market expectations of a change of government can undermine the effectiveness of a policy-maker fully committed to the fixed exchange regime. Hence sovereign spreads can arise. This paper provides an explicit pricing of such risk when political instability is given exogenously.

The Argentine crisis of 2001 is used to illustrate the argument. The country was in a fixed exchange rate regime with a policy-maker committed to not default: Mr Cavallo. Nevertheless, during 2001 the country suffered high country risk and a deep financial crisis. This paper explains why a government fully committed to maintaining a peg coexisted with high country risk.

The paper is organised in three sections. The next section introduces the basic model. Using backward induction, section 3 describes the behaviour of the optimising policy-maker under the succeeding government and its consequent country risk premium. Section 4 reports in which way this premium increases country risk under the first government through the expected (random) switch of government. Finally, we draw some conclusions.

2. The Model

Following Ozkan & Sutherland (1998) we assume that output is determined by global demand conditions, interest rates and the exchange rate. To tailor their model to fit the description of devaluation and default in a highly dollarised economy, we assume that all debts were contracted in US dollars, and all these debts would be pesified after devaluation and default.

Specifically, output is determined as follows:

$$y_t = \alpha\pi + x_t - \gamma s \quad (2.1)$$

where y_t is the output gap (supply minus demand) measured as percentage of GDP, π is the price discount associated with external debt (the country risk), x_t is the global fundamentals (e.g., global slowdown in demand), s is the price of a dollar, all in logs except π . Initially, with one peso to the dollar, s is equal to zero. Output is normalised so that, if there is no

country risk ($\pi_t=0$) and external shocks ($x_t=0$), there will be no output gap at the pegged exchange rate, i.e., demand will match supply.

Let the discount of local dollar debt relative to US equivalent be a proxy for the country risk:¹

$$\pi = c/r - v \tag{2.2}$$

where c is the coupon (measured in \$US) on the unit debt, r is the US interest rate (and so c/r measures the par value of the long maturity debt). The average price of debt is given by $v=V/D$ with D being the fixed amount of the country's debt in dollars and V its value. If the coupon payments of c are expected to be honoured at all times, then with foreign rates constant, the debt price will stand at par (i.e., $v=c/r$); but anticipated reduction of coupon payments (through debt restructuring or default) will lower bond values below par and lead to a country risk premium which affects GNP as bond values are reflected in domestic interest rates.

It is assumed that the country is initially on a fixed exchange rate (where s is normalised to zero). The key exogenous factor driving output is 'global fundamentals' as measured by the variable x_t , assumed to follow a Brownian motion:

$$dx_t = \sigma dZ_t \tag{2.3}$$

where Z_t is a standard Brownian motion and σ is the instantaneous standard deviation. This variable includes effects of world business cycle and the competitive pressures exerted by trade partners: in the Argentinean case, for example, the country was subject to substantial negative shocks due to the slow-down in Latin America, devaluation of the Brazilian Real and the initial weakness of the Euro against the dollar.

If devaluation occurs, a floating exchange rate regime will be adopted. In this case, following Ozkan and Sutherland, it is assumed that the exchange rate acts so as to off-set external shocks. Thus with the floating exchange rate $s=x_t/\gamma$, the last two terms of (2.1) will cancel out. To simplify the treatment, we assume further that (i) no revaluation is possible, and (ii) devaluation will be accompanied by partial default as dollar debt is 'pesified', i.e., converted to peso at devalued rate. With external shocks being stabilised by s and all debts reduced and pesified, country risk will become zero. Hence output will remain at full employment, i.e. $y=0$. We are assuming that an FDR type of policy would have delivered economic recovery.

To capture the experience of a country with high political instability, we introduce the following sequence of events characterising the change of governments. Let the first government be completely committed, and will never choose to devalue because of the high costs it associates with devaluation. The fall of the first government is represented by a Poisson event with an arrival rate of λ per unit time. The probability that the first government loses its power at time t follows an exponential distribution with density function of $\lambda e^{-\lambda t}$. The subsequent government has less commitment to the peg because it has a sma-

1. In the case of partial default, $v = c/r' < c/r$, where r' ($r' > r$) is the effective interest rate in the market when default is anticipated. The country risk is normally defined as $r = r' - r$, and this paper uses $c/r - v = r/(r+r)$, a monotonic transformation of the country risk.

ller perceived cost of devaluation. If the external shock is large, the new government will choose to devalue. Since the first government never devalues, we only consider the devaluation decision under the new government.

We assume that the new government's objective is to minimise expected squared deviations of output from full employment, and that a cost of $C(x)$ is incurred if the government decides to devalue. To capture various different costs of devaluation, we assume in particular that

$$C(x) = F + lx, \tag{2.4}$$

where F indicates a fixed cost and the proportional part, lx , captures the case where perceived costs may be state-dependent, indicating perhaps the difficulties of reaching political consensus and legal agreement after the devaluation.

Since the floating exchange rate regime is assumed to restore output to its full employment level, the output losses after devaluation will be zero. Under these conditions, the loss function of the new government is specified as

$$W(x_t) = \min_{\tau} E_t \left\{ \int_t^{\tau} y^2(x_s) e^{-\rho(s-t)} ds + e^{-\rho(\tau-t)} C(x_{\tau}) \right\}. \tag{2.5}$$

where x_t indicates initial shocks, τ the time for devaluation, ρ the new government's time preferences and E_t the expectations operator, conditional on time t .

In what follows, we first study the behaviour of interest rates (and so country risk) and national output under the second government given the decision to leave an exchange rate peg when external shocks reach a critical level of x_E , known to the markets. Then, there follows the 'political economy analysis' where the decision to leave is made by optimising policy-makers who care about output stabilisation, subject to a time consistency constraint. In all cases, we assume that the decision to abandon the fixed rate regime is irreversible and involves a cost specified in (2.4). In section 4, we look at how anticipated devaluation and default under the second government can generate country risk premium under the current ruling government even if it is fully committed to the peg.

3. Devaluation and Default

3.1. Country Risk under Fully Anticipated Devaluation

Under the second government, devaluation occurs at a pre-determined external shock trigger at x_E , and after the collapse of the peg, one dollar of debt is converted into one peso. Let η indicate the reduction in the par value of the debt in the event of devaluation. At the trigger x_E , devaluation is given by $s(x_E) = x_E / \gamma$, then the debt is reduced to $\eta(x_E) = e^{-s(x_E)}$ of its par value. If $x > x_E$, the devaluation and debt reduction are simply given by $s(x) = x / \gamma$ and $\eta(x) = e^{-x}$.

Let the average debt price v be a function of global fundamentals, x_t . The arbitrage condition for v implies

$$\frac{E_t dv(x_t)}{dt} + c = rv(x_t). \tag{3.1.1}$$

Applying Ito's lemma to (3.1.1) yields the following 2nd order ordinary differential

equation

$$\frac{1}{2}\sigma^2 v''(x) + c = rv(x), \tag{3.1.2}$$

which permits a general symmetric solution

$$v(x) = c/r + A_1 e^{\zeta x} + A_2 e^{-\zeta x}, \tag{3.1.3}$$

where $\zeta = \sqrt{2r/\sigma^2}$ and A_1 and A_2 are constants to be determined.

Since devaluation is irreversible, a value matching condition is required for the price of the debt at the devaluation trigger

$$v(x_E) = \eta(x_E) c/r. \tag{3.1.4}$$

As no revaluation is possible, debt value will approach its par as for favourable fundamentals, i.e. $x \rightarrow -\infty$,

$$\lim_{x \rightarrow -\infty} v(x) = c/r. \tag{3.1.5}$$

Applying (3.1.4) and (3.1.5) to (3.1.3) yields

$$v(x) = \begin{cases} \frac{c}{r} [1 - (1 - e^{-x_E/\gamma}) e^{\zeta(x-x_E)}], & \text{if } x \leq x_E; \\ \frac{c}{r} e^{-x/\gamma}, & \text{if } x > x_E. \end{cases} \tag{3.1.6}$$

The above equation shows that the devaluation trigger x_E has two opposite effects on the price of debt when $x \leq x_E$: the default effect represented by the term $(1 - e^{-x_E/\gamma})$ and the discounting effect by $e^{\zeta(x-x_E)}$. Given an initial x , higher x_E leads to larger devaluation and so a larger reduction in debt value, but higher x_E also implies that it takes longer to reach this trigger, resulting in a higher discounting of such reduction.

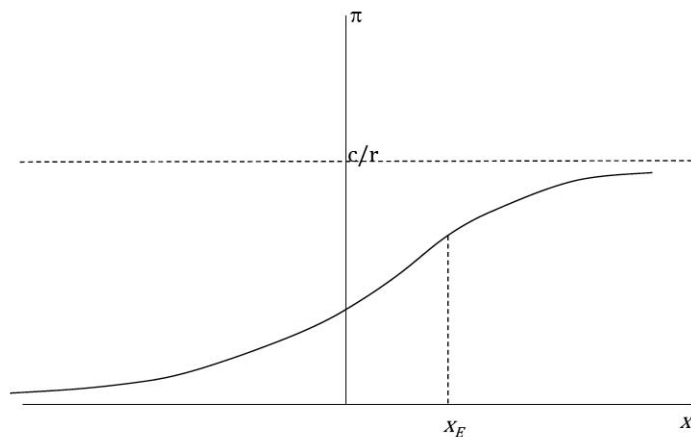


Figure 1: Adverse fundamentals and the price discount

Given devaluation and default occurring at x_E , (3.1.6) and (6.2) determine the country risk under the peg:

$$\pi(x) = \begin{cases} \frac{c}{r} (1 - e^{-x_E/\gamma}) e^{\zeta(x-x_E)} & \text{for } x \leq x_E; \\ \frac{c}{r} (1 - e^{-x/\gamma}) & \text{for } x > x_E. \end{cases} \tag{3.1.7}$$

From (3.6) and (2.1), the resulting output gap is given by

$$y(x) = \begin{cases} \alpha \frac{c}{r} (1 - e^{-x_E/\gamma}) e^{\zeta(x-x_E)} + x, & \text{for } x \leq x_E; \\ 0, & \text{for } x > x_E. \end{cases} \quad (3.1.8)$$

When the devaluation (revaluation) trigger is given, the debt valuation function derived above is shown as an inverted S-shape curve *SS* in Figure 1 where *x* is measured on the horizontal axis. As *x* goes above zero, the country risk premium increases sharply. At the point of devaluation (and revaluation), value matching conditions apply. So, dollar bonds which are to be pesified at a rate of 2 pesos to the dollar on devaluation, for example, will fall to half their par value as *x* approaches *x_E*.

3.2. Time Consistent Devaluation and Default

The time consistent devaluation and default trigger under the second government is determined as follows: given public expectations of devaluation and default at *x_E*, the government chooses its trigger *x_Q* so as to minimise the losses of (2.5) subject to the cost of abandoning the peg (2.4); then, the time consistent equilibrium is obtained when *x_Q* = *x_E* is imposed.

For *x* ≤ *x_Q*, the Feynman-Kac formula implies that the loss function *W(x)* in (2.5) is a solution to the following ordinary differential equation

$$\frac{1}{2} \sigma^2 W''(x) + y^2(x) = \rho W(x), \quad (3.2.1)$$

which permits the general solution

$$W(x) = B_1 e^{\xi x} + B_2 e^{-\xi x} + \frac{(a\gamma)^2}{\rho - 4r} (1 - e^{-x_E/\gamma}) e^{2\zeta(x-x_E)} + \frac{2a\gamma}{\rho - r} (1 - e^{-x_E/\gamma}) \left(x + \frac{\zeta \sigma^2}{\rho - r} \right) e^{\zeta(x-x_E)} + W_N(x), \quad (3.2.2)$$

where $a = ac/(r\gamma)$, $\xi = \sqrt{2\rho/\sigma^2}$, *B₁* and *B₂* are two constants to be determined (assuming $\rho \neq r$ and $\rho \neq 4r$). In the absence of devaluation and default, country risk disappears and the losses are simply given by $W_N(x) = x^2/\rho + \sigma^2/\rho^2$.

To determine *B₁* and *B₂*, two things are worth noting: first, that no revaluation is allowed (*x_Q* ≥ 0), and second that the trigger *x_Q* is optimally chosen. No revaluation implies an asymptotic condition of

$$\lim_{x \rightarrow -\infty} W(x) = \lim_{x \rightarrow -\infty} W_N(x). \quad (3.2.3)$$

This requires *B₂* = 0. No revaluation also implies two distinct cases for the optimal trigger *x_Q*: either *x_Q* has an interior solution of *x_Q* > 0, or *x_Q* = 0. In the case of an interior solution, irreversibility of the decision to float and the optimality of the trigger *x_Q* imply the following value matching and smooth pasting conditions (Dixit & Pindyck 1994)²:

$$\begin{aligned} W(x_Q) &= F + lx_Q, \\ W'(x_Q) &= l. \end{aligned} \quad (3.2.4)$$

2. Note that value matching and smooth pasting conditions are necessary but not sufficient for the interior solution to be a Nash equilibrium. We show this formally in Appendix A.

Eliminating B_1 and imposing time consistency yield an equilibrium trigger x_E as a solution to the equation

$$\phi(x_E) \equiv \frac{2a\gamma}{\sigma^2} (1 - e^{-x_E/\gamma}) \left[\frac{a\gamma}{\xi + 2\zeta} (1 - e^{-x_E/\gamma}) + \frac{2x_E}{\xi + \zeta} - \frac{2}{(\xi + \zeta)^2} \right] + \frac{\xi x_E^2}{\rho} - (\xi l + 2/\rho)x_E = K, \quad (3.2.5)$$

where $K = \xi(F - \sigma^2/\rho^2) - l$. In the case of no interior solution (so $x_Q = 0$), one imposes only (3.2.4).

The following propositions characterise the equilibrium triggers of devaluation and default for differing parameter restrictions.

Proposition 1

Let $K^* = \min_x \phi(x)$. If $a + \ln a \leq 5 - \ln 2 + 1/[\gamma(\xi + \zeta)]$ then:

- (1) for $K < K^*$, $x_E = 0$;
- (2) for $K > K^*$, there is unique consistent devaluation and default trigger $x_E > 0$.

This trigger has the comparative static property $x_E / F > 0$.

Proof: see Appendix A.

Given that external debt has a very long maturity (c/r close to 1), Proposition 1 characterises cases where the effect of country risk on output can be up to at least more than four times larger than that of the exchange rates. For cases where country risk effect is even larger, we have the following proposition.

Proposition 2

If $\rho l/2 + 1/\xi \leq (a + \ln a + 9/4 + \ln 2)\gamma$, time consistent triggers have the same characterization as in Proposition 1.

Proof: see Appendix B.

Relaxing restriction imposed in Proposition 1, Proposition 2 suggests that the uniqueness of the devaluation and default trigger can still be retained as long as the proportional cost for floating the exchange rate is not excessive. Although parameter restrictions imposed in both Propositions 1 and 2 are quite reasonable, they do require that costs associated with devaluation and default are moderate. For the case of extremely high costs, we have the following proposition.

Proposition 3

For cases other than those described in Proposition 1 and 2, there may exist two time consistent devaluation and default triggers. Let $K_1 < K_2$ be the two local minimum, and K_3 be the local maximum of $\phi(x_E)$, then

- (1) for $K < K_1$, $x_E = 0$;
- (2) for $K_2 < K < K_3$, there is a unique equilibrium x_E with x_E / F

Proof: see Appendix C.

Results in Proposition 3 very much resemble those in Obstfeld (1996). When the cost of floating is small, devalue and default at a first possible instance. When the cost of floating is large, normal option value implies a delayed devaluation and default. Multiple equilibria occur when the cost is intermediate. In this case, expectations of early floating reduce the option value of delay and result in actual early devaluation; and similarly for expectations of late devaluation.

4. Country Risk Premium and the Peso Problem

Note that the political instability and expectations of a subsequent new government to devalue, if output gap is sufficiently large, may drive up country risk premium under the current government. By taking as given the subsequent government's decision to devalue and default, we derive country risk under the current 'tough' government and assess how political instability can impact on such premium.

Let t be the random time at which the current government is taken over by a subsequent 'weak' government who will devalue. Following the assumptions made in section 2, the current value of dollar debt is

$$u(x) = E_\lambda E_Z \left\{ \int_0^t c e^{-rs} ds + e^{-rt} v(x_t; x_E) \right\}, \quad (4.1)$$

where $v(x_t; x_E)$ is the value of debt under the new government (as in (3.1.6)), E_Z is the expectations operator over the Brownian motion and E_λ the expectations operator over take-over random time t . The first term represents the discounted coupon payments under the current government, and the second the discounted debt value when the new government takes over.

For an initial external shock of $x(0)=x$, (2.3) has a solution $x_t = x + \sigma Z_t$ where Z_t is normally distributed with mean zero and variance t . Given t follows an exponential distribution with density of $\lambda e^{-\lambda t}$, we can rewrite (4.1) as

$$u(x) = \frac{c}{\lambda + r} + E_\lambda E_Z e^{-rt} v(x + \sigma Z_t; x_E). \quad (4.2)$$

The expected coupon payments under the current government are simply discounted by an effective rate which incorporates the probability that the current government can fall. We relegate the computation of the second term in Appendix D.

Let $\hat{Z} = (x_E - x) / \sigma$, one can show that the debt price is given by

$$u(x) = \begin{cases} \frac{c}{r} \left[1 - \frac{\lambda}{2(\lambda+r)} g_1(x_E) e^{-\sqrt{2(\lambda+r)}\hat{Z}} - (1 - e^{-x_E/\gamma}) e^{-\zeta\sigma\hat{Z}} \right] & \text{if } \hat{Z} > 0, \\ \frac{c}{r} \left[\frac{r}{\lambda+r} + \frac{\lambda}{2(\lambda+r)} g_2(x_E) e^{\sqrt{2(\lambda+r)}\hat{Z}} + \frac{4\gamma^2(\lambda+r)e^{-x/\gamma}}{2\gamma^2(\lambda+r) - \sigma^2} \right] & \text{if } \hat{Z} \leq 0; \end{cases} \quad (4.3)$$

where $g_1(x_E) = 1 - (1 - e^{-x_E/\gamma}) / (1 - \zeta\sigma / \sqrt{2(\lambda+r)}) - e^{-x_E/\gamma} / [1 + \sigma / (\gamma \sqrt{2(\lambda+r)})]$ and $g_2(x_E) = 1 - (1 - e^{-x_E/\gamma}) / (1 + \zeta\sigma / \sqrt{2(\lambda+r)}) - e^{-x_E/\gamma} / [1 - \sigma / (\gamma \sqrt{2(\lambda+r)})]$. The resulting country risk (price discount) is

$$\pi'(x) = \begin{cases} \frac{c}{r} \left[\frac{\lambda}{2(\lambda+r)} g_1(x_E) e^{-\sqrt{2(\lambda+r)}\hat{Z}} + (1 - e^{-x_E/\gamma}) e^{-\zeta\sigma\hat{Z}} \right] & \text{if } \hat{Z} > 0, \\ \frac{\lambda}{(\lambda+r)} \frac{c}{r} \left[1 - \frac{1}{2} g_2(x_E) e^{\sqrt{2(\lambda+r)}\hat{Z}} - \frac{4\gamma^2(\lambda+r)e^{-x/\gamma}}{2\gamma^2(\lambda+r) - \sigma^2} \right] & \text{if } \hat{Z} \leq 0; \end{cases} \quad (4.4)$$

where the first row represents country risk if the current fundamental is below the trigger of the second government, otherwise the country risk is given by the second.

With no political instability ($\lambda=0$), (4.3) shows that debt price is at par (c/r), and (4.4) gives country risk premium of zero. The presence of political instability reduces the price and increases country risk. When the change of the government is immediate ($\lambda \rightarrow \infty$), country risk under the first government is identical to that of the second (as (4.3) degenerates to (3.1.6) and (4.4) becomes (3.1.7)).

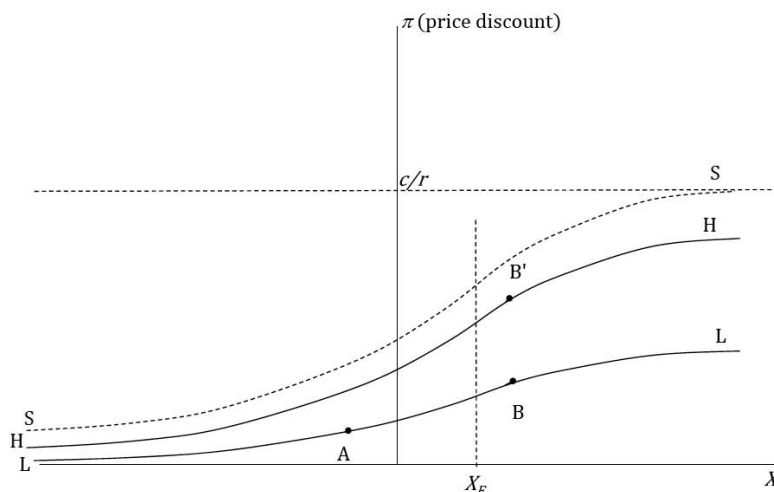


Figure 2 : Political instability and the peso problem

Figure 2 illustrates how country risk under the first government is determined (the horizontal axis is the fundamental while the vertical is the country risk). Dashed curve SS represents country risk under the second government (it is also the one for the first government if $\lambda \rightarrow \infty$). The horizontal axis gives the country risk for the first government if $\lambda=0$. For any given $\lambda > 0$, country risk for the first government is simply a weighted average of SS and the horizontal axis (where the weight is state dependent). Two possible country risk profiles for the first government are drawn: HH corresponds to $\lambda = \lambda_H$ and LL to $\lambda = \lambda_L < \lambda_H$.

Can this model account for the country risk premium evolution in Argentina before the collapse of the Convertibility regime? Figure 2 represents the relatively low country risk of 10% (mid-2001) by point A which corresponds to low political risk. Deteriorating fundamentals could account for the gradual increase of country risk (it rose to 15% in the third quarter), as indicated by the movement from A to B' . It is clear that, towards the end of 2001, the market revised upwards the exit probability of de la Rúa's regime. Interpreting this as an increase in the parameter λ , this will shift the price-discount schedule up from LL to HH . This corresponds to a rapid increase in country risk (it jumped to 40% in the fourth quarter) before the fall of de la Rúa's government.

3. Where it is assumed that the fundamental has passed the trigger for devaluation and default of the second government, labeled x_E .

Conclusions

Market expectations of a change of government can undermine the effectiveness of a policy-maker fully committed to the fixed exchange regime. Hence sovereign spreads can arise. This paper provided an explicit pricing of such risk when political instability is given exogenously.

Firstly, the paper presented the basic model. Secondly, it described the behaviour of the new government and its country risk premium associated. Finally, assuming an exogenous given expected switch of government, the paper reported how the expected ex-post country risk increases the ex-ante risk.

There are possibilities for interesting ways forward through endogenising the political uncertainty (parameter λ) into the model⁴. This would demand a more complex framework but would allow the model to account for the influence of bad fundamentals on the probability of change of government. Moreover, it would give a more detailed description for the Argentine collapse.

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4. Another approach, suggested by the anonymous referee, is to assume that λ is a time varying stochastic process but exogenous

Appendices

A. Proof of Proposition 1

We show first that the function $\phi(x_E)$ is strictly convex under the given assumptions. Using the fact that ϕ is initially strictly decreasing and then increasing, we establish interior time consistent solutions and their comparative static properties. Finally, we show that $x_E = 0$ if the devaluation and default cost is relatively low. For a simple exposition, we drop the subscript of x .

For $\phi'(x) > 0$, it is equivalent to have

$$\varphi(x) \equiv \frac{\sigma^2 e^{x/\gamma}}{4a} \phi''(x) = \frac{2ae^{-x/\gamma}}{\xi + 2\zeta} + \frac{e^{x/\gamma}}{a\xi} - \frac{x}{\gamma(\xi + \zeta)} + \frac{1}{\gamma(\xi + \zeta)^2} + \frac{2}{\xi + \zeta} > 0. \quad (6.1)$$

Function $\varphi(x)$ is strictly convex and has a unique minimum at

$$x^* = -\gamma \ln(a^*/a), \quad \text{where} \quad a^* = \frac{\xi + 2\zeta}{4(\xi + \zeta)} \left(\sqrt{1 + \frac{8(\xi + \zeta)^2}{\xi(\xi + 2\zeta)}} - 1 \right) \quad (6.2)$$

If $a \leq a^*$, $\phi(x)$ is strictly convex. This is because $x^* \leq 0$, and $\varphi(x)$ is strictly increasing for $x \geq 0$. Since $\varphi(0) > 0$, so $\varphi(x) > 0$ for $x \geq 0$.

If $a > a^*$, $x^* > 0$. Strict convexity of $\phi(x)$ now requires $\varphi(x^*) \geq 0$. This translates into the following parameter restriction

$$\ln a + \frac{\xi + \zeta}{\xi + 2\zeta} a \leq \ln a^* + \frac{4(\xi + \zeta)}{\xi + 2\zeta} a^* + 3 + \frac{1}{\gamma(\xi + \zeta)}. \quad (6.3)$$

The first two terms on the RHS of (6.3) are decreasing in ζ and have a minimum of $2 - \ln 2$ when $\zeta \rightarrow \infty$. So $\ln a + a \leq 5 - \ln 2 + 1/[\gamma(\xi + \zeta)]$ is sufficient for (6.3).

Since $\phi(x)$ is strictly convex, with $\phi'(0) < 0$ and $\phi'(\bar{x}) > 0$, $\phi(x)$ must have a unique minimum $K^* = \phi(\bar{x})$ and $\bar{x} > 0$. So $\phi'(x) < 0$ for $x \in [0, \bar{x})$, and $\phi'(x) > 0$ for $x > \bar{x}$.

For $K^* < K < 0$, as $\phi(0) = 0$ and $\phi(\bar{x}) = K^*$, there must be two solutions: $0 < x'_E < \bar{x}$ and $x_E > \bar{x}$. We show that x'_E is not a Nash equilibrium while x_E is. Note that from loss function given in (3.2.2) a minimum of W is equivalent to a minimum of B_1 (as $B_2 = 0$). Using (3.2.4) to solve for B_1 yields

$$B_1 = e^{-\xi x_Q} \left[F + lx_Q - \frac{(a\gamma)^2}{\rho - 4r} (1 - e^{-x_E/\gamma})^2 e^{2\zeta(x_Q - x_E)} - \frac{2a\gamma}{\rho - r} (1 - e^{-x_E/\gamma}) e^{\zeta(x_Q - x_E)} \left(x_Q + \frac{\zeta\sigma^2}{\rho - r} \right) - \frac{x_Q^2}{\rho} - \frac{\sigma^2}{\rho^2} \right]. \quad (6.4)$$

Differentiating B_1 with respect to x_Q and imposing the time consistency $x_Q = x_E$ gives

$$\frac{\partial B_1}{\partial x_Q} \Big|_{x_Q=x_E} = e^{-\xi x_E} [\phi(x_E) - K], \quad (6.5)$$

where at x'_E , $\phi(x'_E) - K = 0$. The strict convexity of $\phi(x)$ implies $\phi'(x'_E) < 0$. Consider a small reduction of x_Q from x'_E (while by still imposing time consistency), this leads to an increase in B_1 and so the loss function. Thus x'_E is not a Nash equilibrium. Using this similar local argument, one can show that x_E is a Nash equilibrium.

To establish the comparative static property of x_E , note that $\phi(x)$ is locally increasing at x_E , so $x_E/F > 0$.

For $K > 0$, there is only one solution x_E satisfying $\phi(x_E) - K = 0$. As $\phi(x_E)$ is locally increasing, so x_E is a Nash equilibrium. Similarly, we also have $x_E/F > 0$.

If $K < K^*$, there is no interior solution. In this case, only (3.2.4) can be imposed as a boundary condition. This results B_1 as in (6.4) and its derivative with respect to the trigger as in (6.5). For $K < K^*$ and any $x_E > 0$, $B_1/x_Q|_{x_Q=x_E} > 0$, so $x_E = 0$.

B. Proof of Proposition 2

Here we only need to show that $\phi(x)$ is initially decreasing and then increasing for $x \geq 0$ under the given parameter restriction. The rest of the proof follows directly from Appendix A.

Let

$$\psi(x) \equiv \frac{\sigma^2 e^{x/\gamma}}{4a} \phi'(x) = \frac{a\gamma}{\xi + 2\zeta} (1 - e^{-x/\gamma}) + \frac{x - \hat{x}}{a\xi} e^{x/\gamma} \quad (7.1)$$

$$+ \frac{x}{\xi + \zeta} - \frac{1}{(\xi + \zeta)^2} - \frac{\gamma}{\xi + \zeta}$$

where $\hat{x} = \rho l/2 + 1/\zeta - a\gamma\zeta/(\zeta + \xi)$. Differentiating ψ yields

$$\delta(x) \equiv e^{-x/\gamma} \psi'(x) = \frac{ae^{-2x/\gamma}}{\xi + 2\zeta} + \frac{e^{-x/\gamma}}{\xi + \zeta} + \frac{x - \hat{x} + \gamma}{a\gamma\xi} \quad (7.2)$$

So $\psi(x)$ is strictly increasing as long as $\delta(x) > 0$ for $x \geq 0$.

Function $\delta(x)$ is strictly convex and has a unique stationary point at x^* (as defined in Appendix A), so $\delta(x^*)$ is the minimum. If $a \geq a^*$, $x^* > 0$. Positive $\delta(x)$ requires $\delta(x^*) > 0$, which in turn imposes the following parameter restriction

$$\frac{1}{2}\rho l + \frac{1}{\xi} \leq \left[\frac{\xi a}{\xi + \zeta} + \ln a + \frac{\xi}{2(\xi + \zeta)} a^* - \ln a^* + 2 \right] \gamma \quad (7.3)$$

$$\leq (a + \ln a + 9/4 + \ln 2)\gamma$$

So given (7.3), $\psi(x)$ is strictly increasing in x for $x \geq 0$. As $\psi(0) < 0$ and $\psi(+\infty) > 0$, $\psi(x) = 0$ has a unique solution \bar{x} , and $\psi'(x) > 0$ for $x \in [0, \bar{x}]$, $\psi'(x) < 0$ for $x > \bar{x}$.

C. Proof of Proposition 3

It is obvious from (7.1) that, for $x \geq 0$, $\phi(x)$ is initially convex, then concave, and finally convex. At the most $\phi(x)$ can have two local minima $K_1 < K_2$ and one local maximum K_3 . For $K_2 < K < K_3$, $\phi(x) - K = 0$ has four zeros: two of which occur when $\phi'(x) < 0$ (so ruled out for Nash equilibria) and the other two with $\phi'(x) > 0$ (so they constitute Nash equilibria). The rest of the proof follows exactly as in Appendix A.

D. Country Risk Premium under the First Government

Denote the second term in (4.3) by I , then

$$\begin{aligned} I &\equiv E_\lambda E_Z e^{-rt} v(x + \sigma Z_t; x_E) \\ &= \int_0^\infty \lambda e^{-\lambda t} \left[\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi t}} e^{-s^2/(2t) - rt} v(x + \sigma s; x_E) ds \right] dt \\ &= \frac{2\lambda}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} v(x + \sigma s; x_E) \left[\int_0^\infty e^{-(\lambda+r)(\sqrt{t})^2 - (1/2)s^2/(\sqrt{t})^2} d\sqrt{t} \right] \end{aligned} \quad (9.1)$$

Using the formula

$$\int_0^\infty e^{-ax^2 - b/x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}, \quad \text{for } a > 0 \text{ and } b > 0$$

(Gradshteyn and Ryzhik, 1994, 3.325, p355), (9.1) becomes

$$\begin{aligned} I &= \frac{\lambda}{\sqrt{2(\lambda+r)}} \int_{-\infty}^{+\infty} v(x + \sigma s; x_E) e^{-\sqrt{2(\lambda+r)}|s|} ds \\ &= \frac{\lambda}{\sqrt{2(\lambda+r)}} \frac{c}{r} \left\{ \int_{-\infty}^{\hat{Z}} \left[1 - (1 - e^{-x_E/\gamma}) e^{\zeta(x + \sigma s - x_E)} \right] e^{-\sqrt{2(\lambda+r)}|s|} ds \right. \\ &\quad \left. + \int_{\hat{Z}}^{+\infty} e^{-(x + \sigma s)/\gamma} e^{-\sqrt{2(\lambda+r)}|s|} ds \right\} \end{aligned} \quad (9.2)$$

Some straightforward integrations for different cases of \hat{Z} yield (4.3) in the text.