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SAVING DECISIONS UNDER BOUNDED AND HETEROGENEOUS SOPHISTICATION

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ABSTRACT

We consider saving decisions in an economy where agents are characterized by heterogeneous levels of sophistication. In our simple setting, we are able to make a distinction between aspects of fundamental sophistication and strategic sophistication. The equilibrium of this economy is inefficient as sophisticated households do not offset the distortions that result from information and behavior of naive households. We show instances in which increments in the ability of sophisticated households to perceive more precise information results in less efficient outcomes. Additionally, we identify simple policy interventions that would increase welfare levels.

Key Words: saving decisions, heterogeneity, sophistication.

RESUMEN

Se estudia una economía en que agentes dotados de diverso grado de sofisticación deciden el nivel de ahorros. En el simple marco de análisis propuesto, es posible hacer una distinción entre sofisticación estructural y sofisticación estratégica. El equilibrio de esta economía es ineficiente debido a que el accionar de los hogares con alto nivel de sofisticación no logra corregir las distorsiones que resultan de las creencias y el comportamiento asociado los hogares menos sofisticados. Se muestran ejemplos en los que incrementos en la capacidad de los hogares sofisticados para percibir información más precisa resulta en asignaciones menos eficientes. Se identifican intervenciones de política simples que resultan en mayores niveles de bienestar.

Palabras Clave: decisiones de ahorro, heterogeneidad, sofisticación.

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I. Introduction

When making saving decisions, households use explicit or implicit assessments of the future flow of income. These evaluations are formed while interacting in an economy populated by other households solving similar problems. If the understanding of the environment is complete then, the associated decisions will respond with precision to future income flows; if this is the case, there is little value in evaluating the process through which these accurate expectations are formed. Additionally, in the absence of externalities, the allocations that result from competitive markets would be efficient. In contrast, once limited and asymmetric predictive abilities are admitted, there is value in evaluating the link between alternative profiles of predictive abilities and economic outcomes.

In this spirit, the present work focuses on a simple situation in which agents are characterized by different capacities with respect to their ability to make forecasts. More specifically, we consider a two period endowment economy populated by households that interact in a market where they exchange loan contracts. Households are characterized by different levels of sophistication. *Naïve* (naive) households make decisions influenced by non-informative signals and respond to prices. Sophisticated households make decisions considering prices and informative private signals of future income. In addition, sophisticated households infer information from market dynamics. Interpreting the information contained in market outcomes requires a model of the way other agents behave and information regarding others agents' beliefs. We assume sophisticated households are not unlimitedly so; their skills are constrained by the signals' precision and the accurateness of their models of others'.

The equilibrium of this economy is in general inefficient due to the joint influence of imperfect information, inadequate decisions given information sets and imperfect substitutability in consumption that limits the capacity to offset distortions. Inefficiencies persist even in instances when sophisticated households' beliefs are correct. In this case, sophisticated households accommodate *naïve* demand shock through a combination of variation in prices and trades in financial contracts.

Our analysis illustrates how changes in different aspects of sophisticated households' learning processes are associated with variations in actions, price patterns and welfare levels. We show instances in which increments in specific skills of sophisticated households can result in lower welfare levels. For example, increasing the precision with which sophisticated households observe *naïve* households' beliefs can result in lower welfare. Additionally, welfare can decrease as sophisticated households perceive more precise information regarding future income. The message seems to indicate that under the presence of various imperfections in the ability to learn about the environment and select proper actions, improvement in some learning skills that are not accompanied by improvements in other aspects might lead to worse outcomes. This type of result can be explained as one instance in which the theory of the second best applies.[1] That is, in an economy with multiple imperfections reducing one of those imperfections does not necessarily lead to better outcomes.

The present work focuses on a situation in which the description of the way in which the agents form expectations is a relatively complex object. On the other hand, the structure of the economy is extremely simple. In particular, this is a scenario it is not possible to modify aggregate consumption levels. The interpretations of the results provided and the determination of its range of validity need to be made taking into account these considerations.

Our approach is related to contributions in behavioral finance that develop theoretical frameworks in which agents with different levels of sophistication interact. De Long, Summers and Waldmann (1990a) is a prominent contribution that models a financial market in which one fraction of the investors hold biased beliefs regarding the return of the traded assets. They show that short term sophisticated agents are not able to eliminate the distortions that result from noise trader's price pressure. In a related paper, De Long *et al.* (1990b) analyze a scenario in which sophisticated arbitrageurs can anticipate noise trader's reaction to past prices. In this context, sophisticated agents are responsible for excess volatility as they trade anticipating that the reaction of *naïve* traders to this trading activity will permit unloading the positions at a gain.

1. *For the seminal paper see Lipsey et al. (1956).*

In the macroeconomics literature, Haltinwanger *et al.* (1985 and 1989) studies environments with strategic complementarities in which there exist heterogeneity in abilities to form expectations. These analyses indicate that, given complementarities, less sophisticated agents can have a large impact as sophisticated agents select actions that do not offset but reinforce the initial impact of *naïve* choices. In contrast to these contributions, the present analysis does not allow for agents which unlimited prediction abilities. In addition, this paper studies the impact of changes in skills and policies in a context with no strategic complementarities.

Our contribution is also related to behavioral game theory analyses that study scenarios in which players' strategic sophistication is limited. That is, players are not able to fully understand the incentives of others' and hence might not be able to correctly anticipate their actions or the information contained in their actions. These analyses typically allow for outcomes that, from the perspective of traditional analysis, can be termed as out of equilibrium (see for example Crawford, 2004; Camerer *et al.*, 2004; Esponda, 2008; and Eyster and Rabin, 2005).

The next section presents the model and analyzes properties of the equilibrium. Section III evaluates scenarios in which the skills of sophisticated households change. Section IV considers some policy implications that result from this analysis. Section V present discussions regarding this exercise and evaluates potential extensions.

II. Model

In this section we introduce a simple model of saving decisions under heterogeneous levels of skills. Consider a two period model in which households' preferences are represented by a separable utility function $u(c_{1i}, c_{2i}) = u(c_{1i}) + u(c_{2i})$. Where c_{ji} represents quantity consumed by household i of the unique good of the economy in period j. In both periods, income across households is the same and there is no storage technology. First period income, w_1, is known but second period income, w_2, is uncertain with distribution $N(\bar{w}, \sigma_w^2)$. For each period, all households earn the same income level.

Borrowing and lending activity is permitted through a financial market. In this market, households exchange units of period 1 goods for contracts in which the issuer promises to transfer the holder units of period 2 goods. For simplicity, it can be assumed that there is a central counterparty that takes one side in every transaction. The interest rate R is equal to the quantity of period 2 goods conceded in a contract that is sold for one unit of period 1 good. This interest rate is determined by the market clearing condition. Let l_i represent the quantity borrowed by household i, or the negative of the quantity lent by household i. In our simple setting, each agent selects, for a given level of the interest rate R, the level of borrowed/lent funds, l_i. The associated budget constraints are given by:

$$c_{1i} = w_1 + l_i$$
$$c_{2i} = w_2 - Rl_i$$

Heterogeneity will enter through differences in the way household understand their environment. We consider an economy populated by two types of households: *naïve* and sophisticated. We assume that there is a continuum of measure 1 of each type of agent. The only distinction between households will be associated with skills through which they learn about future income. Assuming the existence of two groups with very different skills is a simplification that allows for a more clear understanding of the outcomes associated with the model. Similar representations with less strict assumptions would lead to qualitatively similar results.

We assume each household selects a savings level according to a simple rule. Given their beliefs about future income, both types of households select the amount of savings or borrowings as a function of the expected income, current income and the interest rate. A reasonable rule would result in higher borrowings in response to an increase in expected future income, a decrease in current income or a raise in the interest rate. We postulate that the amount borrowed equals: $l(E_i [w_2], w_1, R) = 1/2 \{(E_i [w_2] / R)- w_1\}$. Where $E_i [w_2]$ is the expected value of income in period 2 according to agent i's beliefs.

That is, borrowings equal one half of the difference between the present value of future income and current income. This rule satisfies the property that the associated consumption profile responds to the relative price of consumption in each period, and the present value of wealth. It can be rationalized by observing that this would be the choice made in the certainty equivalence case by an agent with logarithmic preferences. In essence, this rule captures a tradeoff between gains from responding to price and losses from asymmetric consumption profile. These tradeoffs are evaluated in expected values of consumption profiles, that is, abstracting from uncertainty. As a result, this rule can be seen as a form of overconfident behavior. The interpretation of this work can be made based on this premise.

II.1 *Naïve* Households

Naïve households receive a signal of the second period income $n_i = n + u_i$ where n and u_i are iid random variables distributed normally according to $N(\overline{n},\sigma_n^2)$ and $N(0,\sigma_u^2)$ respectively. That is, we assume these households receive uninformative correlated signals and make decisions that are influenced by these signals. Given the simple rule proposed as a characterizing households' behavior the demand for loans is given by:

$$l(n_i, R) = \frac{n_i - Rw_1}{2R}$$

Their characterization involves two types of *naïveté*, one is fundamental and deals with the lack of information in the private signals regarding future income; the second is strategic and is given by the absence of any consideration of the information contained in others' choices.

II.2. Sophisticated households

In contrast, the sophisticated households receive unbiased but noisy signals of their future income flow. The precision of this signal is an aspect of the fundamental sophistication of this type of households. In addition each sophisticated household receives a signal of the average beliefs of *naïve* households. They use this signal jointly with current prices to infer information about future income. Their strategic sophistication is captured by this process. Note that inferring information from prices requires having a model of how different information sets of *naïve* and sophisticated households lead to different actions and associated prices. As a result, the capacity to infer information through this process is bounded by the mistakes of the signal regarding *naïve* beliefs and the shortcomings of the models regarding how prices are formed.

More specifically, sophisticated households receive an unbiased signal of the second period income, $w_{2i}^{s} = w_2 + e_i$ where e_i's are independent and identically distributed random variables with normal distribution given by $N(0,\sigma_e^2)$. In addition, they have initial beliefs on *naïve* expectations given by $n_0 : N(\hat{n},\sigma_{\hat{n}}^2)$ and perceive a signal of *naïve* households' expectations $n^S = n_0 + (\hat{n} - \overline{n}) + v$ with v distributed according to $N(0,\sigma_v^2)$. That is, we allow for an informative but biased signal in addition we allow for errors in the evaluation of the precision of priors beliefs regarding *naïve* expectations. We assume sophisticated households act as if this signal and the beliefs on its precision were correct.

Given R and n^S sophisticated households extract a signal of future income. This signal extraction exercise exploits the fact that, in this economy, there is a positive association between interest rates and average expectations of future income.[2] The problem to be solved involves assessing the impact of noisy households on market prices. This impact is a function of how much does sophisticated households' behavior offset this noisy price pressure. We assume that sophisticated households think all households make financial decisions that are based on a simple use of their private signals on future income. That is, the price signal is extracted assuming that *naïve* decide using n_i as their future income and that sophisticated households make financial decisions using w_{2i}^{S} as their future income. In other words, we assume that sophisticated households' representation of others behavior is a simplification that ignores the fact that other sophisticated households would also infer information from prices and use prior beliefs in their assessment of future wealth.[3] As a result, in this hypothetical exercise, the demand for loans for each type of household is given by:

2. This association is the result of the simplifying assumptions regarding the structure of this economy. In economies with risky contracts and investment this simple association does not need to hold.

3. This assumption is in line with contributions in behavioral game theory that evaluate the existence of bounds in the understanding of other players' strategic responses. Despite common priors and knowledge of the structure of the game, these bounds on strategic reasoning imply that there is no full coordination of expectations and actions selected are out of equilibrium. (see Crawford, 1997).

$$l(n_i;R)=\frac{n_i-Rw_1}{2R} \qquad \text{and} \qquad l(w_{2i}^S;R)=\frac{w_{2i}^S-Rw_1}{2R}$$

Then, the interest rate is thought to be the result of the following market clearing condition:

$$\int_i l(n_i,R)di+\int_i l(w_{2i}^S,R)di=0$$

Replacing the demand functions, integrating across agents and solving for R results in:

$$R=\frac{w_2+n}{2w_1}$$

That is, through their inference process, sophisticated players conclude that there is an affine function that describes the relationship between future income, average *naïve* expectations and the interest rate. In this way, sophisticated players come up with a second signal of future income that is a function of the interest rate and their private signal regarding the expectations of *naïve* households:

$$\hat{w}_{2i}^S=2w_1R-E(n\,|\,n^S)$$

Where:

$$E[n\,|\,n^S]=\left(\frac{\hat{n}}{\sigma_{\hat{n}}^2}+\frac{n^S}{\sigma_v^2}\right)\Bigg/\left(\frac{1}{\sigma_{\hat{n}}^2}+\frac{1}{\sigma_v^2}\right)$$

Under the assumptions made by the household in this signal extraction exercise, conditioning on signal n^S, the expectation of $\hat{w}_{2i}^S$ is w_2, that is, this second signal is thought to be an unbiased predictor of future income. Note that, according to this inference process, deviations of the signal from the true value are the result of errors in the expectations of the average beliefs of *naïve* households. In addition, according to the assumptions of the inference process, the variance of this signal is given by:

$$\sigma_{\hat{w}}^2 = \frac{\sigma_{\hat{n}}^2 \sigma_v^2}{\sigma_{\hat{n}}^2 + \sigma_v^2}$$

In this way, sophisticated households observe two signals, w_{2i}^S and $\hat{w}_{2i}^S$. We assume that, the updated beliefs are formed using Bayes' rule. This assumption result in what has been referred in the literature as quasi-bayesian expectations[4]; that is, Bayesian updating based on mistaken assumptions about the distribution of the signals.[5] Using Bayesian updating, the two signals result in the following updated expectations of future income for sophisticated household *i*:

$$E[w_2 \mid R, n^s, w_{2i}^S] = \frac{\frac{\overline{w}}{\sigma_w^2} + \frac{w_{2i}^S}{\sigma_e^2} + \frac{\hat{w}_{2i}^S}{\sigma_{\hat{w}}^2}}{\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_{\hat{w}}^2}}$$

For this type of household, expected income equals a weighted sum of the prior, a private signals and the public signal inferred from the market equilibrium. The weights are given by the assessment of precision attached to each element. These beliefs are associated to a demand for loans given by:

$$l(R, n^S, w_{2i}^S) = (E[w_2 \mid R, n^S, w_{2i}^S] - Rw_1)/2R$$

II.3. Equilibrium

Thus, we have already determined the behavior of each type of households given the process through which they form beliefs. Given these behavioral responses we can use the market clearing condition together with *naïve* and sophisticated households' expectations to determine the condition that must be met by the equilibrium interest rate. The corresponding mar-

4. See for example models of confirmatory bias (Rabin et al., 1999) and local representativeness (Rabin, 2002).

5. Our qualitative results do not depend on the updating of beliefs through Bayes' rule. We would observe similar outcomes under alternative specifications of weighting of the signals.

ket clearing condition results from setting the sum of the demand functions of sophisticated households and *naïve* households equal to zero:

$$\int_i l(n_i, R)di + \int_i l(E[w_2 \mid R, n^S, w_{2i}^S], R)di =$$
$$= \int_i \frac{E[w_2 \mid R, n^S, w_{2i}^S] - Rw_1}{2R} di + \int_i \frac{n_i - Rw_1}{2R} di = 0$$

Replacing the expressions corresponding to the expectations of each type of household and calculating the integral across households results in an expression that identifies the interest rate in an implicit form:

$$2w_1 R = \int_i E[w_2 \mid R, n^S, w_{2i}^S]di + n =$$
$$= \frac{\frac{\overline{w}}{\sigma_w^2} + \frac{w_2}{\sigma_e^2} + \frac{\left(2w_1 R - \left(\frac{\hat{n}}{\sigma_{\hat{n}}^2} + \frac{n^S}{\sigma_v^2}\right) \Big/ \frac{1}{\sigma_{\hat{n}}^2} + \frac{1}{\sigma_v^2}\right)}{\sigma_{\hat{w}}^2}}{\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_{\hat{w}}^2}} + n$$

In this expression the interest rate appears in the two sides of the inequality. This reflects the dual role that this price plays in this market clearing mechanism. On one hand, the interest rate has a direct negative impact on the demand for loans given by what can be identified as the traditional response of a demand function. The second effect is given by the information contained in prices. Higher prices are identified as signal of higher future income by sophisticated agents, and as a result, this increases the demand for loans. This is reflected by the appearance of *R* on the right hand side of the equation. Replacing $\sigma_{\hat{w}}^2$, solving for *R* and rearranging we get an explicit expression for the equilibrium price in this simple market:

$$R^e = \left[\frac{\frac{\overline{w}}{\sigma_w^2} + \frac{w_2}{\sigma_e^2}}{\left(\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2}\right)} + n + \frac{n - \hat{n}}{\sigma_{\hat{n}}^2 \left(\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2}\right)} + \frac{n - n^S}{\sigma_v^2 \left(\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2}\right)} \right] \frac{1}{2w_1}$$

This expression provides a clear picture of the determinants of the interest rate in this model. The first term inside brackets captures the impact of sophisticated traders' fundamental information. This term gets closer to w_2 as the precision of the signal $\hat{w}_{2i}^S$ gets large, that is, as $1/\sigma_e^2$ increases.

The second term inside brackets, n, captures the direct impact of *naïve* price pressure. Higher beliefs by *naïve* investors drive interest rates up. The last two terms capture the impact of surprises that sophisticated households' suffer regarding the value of *naïve* beliefs. More specifically, the third term captures de deviations with respect to prior beliefs, $\hat{n}$, and the fourth term captures deviations with respect to the signal perceived, n^S. Surprises with a positive sign mean that sophisticated agents attributed, mistakenly, higher interest rates to higher sophisticated agents' fundamental signals. This, in turn, increases sophisticated households' future income expectations and increases their demand for loans. As a result, due to the nature of the inference process, *naïve* beliefs which are higher than expected by sophisticated agents imply higher interest rates. The impact of each type of surprise, with respect to priors or with respect to signals, depends on the beliefs regarding the precision of each piece of information. The higher the precision attributed to the prior, $1/\sigma_{\hat{n}}^2$, the higher the impact of a deviation from the prior, $n-\hat{n}$.

There are similar implications for the link between the impact of deviations of the signal, n-n^S, and the precision of the signal, $1/\sigma_v^2$. Note that the impact of each of these deviations is also a function of the precision of the fundamental prior and the fundamental signal. Higher precision, lower σ_w^2 and σ_e^2, imply a lower impact of the deviations between expected and actual *naïve* beliefs. In the limit, as σ_w^2 or σ_e^2 converge to zero, the impact of their deviation converges to zero. On the other hand, the second component, "n", has an impact that does not depend on the precision of fundamental information. That is, a form of demand pressure that is not offset by informed investors' actions.

To summarize, interest rate increases are the result of multiple factors: increases in future wealth, shocks in *naïve* preferences and mistakes in the priors or the signals regarding *naïve* beliefs.

II.4. Description of equilibrium outcomes

In this subsection we intend to provide simple illustrations of the outcomes associated to this environment. For simplicity we normalize the parameters in this model: $w_1 = \overline{w} = 1$ and set the same value for the variance of the error term in $\sigma_v^2 = \sigma_{\hat{n}}^2 = \sigma_w^2 = \sigma_e^2 = 1$. Then expressions for the equilibrium outcomes of the model simplify to:

$$R^e = \frac{1}{2}\left[\frac{1+w_2}{2} + n + \frac{(2n - \hat{n} - n^S)}{2}\right]$$

$$E[w_2 \mid R^e, n^S, w_{2i}^S] = \frac{1}{2} + \frac{w_{2i}^S + w_2}{4} + \frac{4n - \hat{n} - 3n^S}{4}$$

$$l(n_i, R^e) = \frac{2n_i}{[1 + w_2 + 4n - \hat{n} - n^S]} - \frac{w_1}{2}$$

$$l(E[w_2 \mid R^e, n^S, w_{2i}^S], R^e) = \frac{1 + \frac{w_{2i}^S + w_2}{2} + 2n - \hat{n} - n^S}{[1 + w_2 + 4n - \hat{n} - n^S]} - \frac{w_1}{2}$$

We will consider the outcomes associated to four simple scenarios. We will evaluate the corresponding level of the interest rate and the quantities lent or borrowed by the modal agent of each type. In all these cases we focus on the effect of *n aïve* agent expectational shocks and will assume there are no fundamental shocks, that is $w_2 = \overline{w}$.

The first scenario we consider, "a", is the benchmark case in which there are no surprises. In "a" average *naïve* beliefs are correct, $n = w_2$, and there are no mistakes in the sophisticated beliefs regarding *naïve* expectations, $\hat{n} = \overline{n} = n$. In this scenario the interest rate equals $R^* = 1$, the future income expectations of the modal sophisticated agent are correct and loan levels of the modal agents are zero ($l(n_1, 1) = l(E[w_2 \mid 1, n^S, w_2], 1) = 0$).

Next, we consider a scenario, case "b" in which *naïve* households' average beliefs are higher than future income but sophisticated households anticipate perfectly this shock to beliefs. That is, we consider a case in which

$w_2 < n = \hat{n} = \overline{n}$. In this case, the equilibrium interest rate is higher that the perfect information value: $R^e = 1/2 + n/2 > 1$. But sophisticated households make no mistake, the beliefs of the modal sophisticated household are correct $E[w_2 | R^e, n^s, w_2] = 1$ with resulting borrowed quantities $l(\mathrm{E}\,[w_2 | R^e, n, 1], R^e) = 1/(1 + n) - 1/2 < 0$. Not surprisingly given the market clearing condition, the modal *naïve* household's borrowed quantities are given by $l(n_i, R^e) = n/(1+n) - 1/2 > 0$. Hence, with *naïve* expectations which are overoptimistic but correctly anticipated by sophisticated households, the interest rate would increase and sophisticated households would on average lend to *naïve* households. Sophisticated households accommodate part of the *naïve* price pressure by lending at rate that is higher than what would be observed under perfect information. That is, the impact hits both price and volume.

We can consider two alternatives to the case we have just analyzed. One alternative scenario would be the case in which sophisticated households partially anticipate *naïve* expectations shock. While we could also consider the case in which *naïve* expectations shocks are completely unexpected.

Suppose the case of partial anticipation (case "c"). In this case we assume that prior beliefs indicate that *naïve* households are on average right, that is $w_2 = \overline{n}$; overoptimistic *naïve* beliefs are signaled to sophisticated households by $\hat{n} = n > w_2$. In this case the equilibrium interest rate equals $R^e = 1/4 + 3n/4$; this is higher than the interest rate of the previous example as sophisticated investors attribute part of the demand pressure to higher income in the following period. Sophisticated expectations are given by: $E[w_2 | R^e, n, w_2] = 1/2 + n/2$. Borrowing by a modal *naïve* player equals $l(n_i, R^e) = 2/(3+1/n) - 1/2 > 0$ and the negative of sophisticated lending equals: $l(\mathrm{E}\,[w_2 | R^e, n, 1], R^e) = 1/2 - 2/(3 + 1/n) < 0$. In this case, there is a smaller amount of transactions as sophisticated agents are less willing to lend given their optimistic expectations on future income.

Finally, consider case "d" in which *naïves*' expectations shock is not anticipated by priors or signals. That is, we consider the case in which $w_2 = \hat{n} = \overline{n} < n$. The equilibrium interest rate equals $R^e = n$ this is also equal to the conditioned expectations of income by sophisticated households: $E[w_2 | R^e, 1, w_2] = n$. The loan levels of the modal sophisticated household and the modal *naïve* household equal 0. That is, sophisticated households attribute all the increment in the interest rate to informative fundamental signals, thus, at this high rate, unlike what occurs in the previous examples; they are not willing to make loans. This is the case in which the interest rate deviates the most from what would be observed under perfect information. At the same time, the consumption path of the modal players coincides with what would be observed under an efficient allocation. As a result, mistakes in expectations blocks transactions that would have reduced the efficiency of the equilibrium allocation. This equilibrium could be interpreted as a situation in which there is no trade between *naïve* and sophisticated households, the only trade in the market for loans can be attributed to trades between agents of the same type that receive different signals.

Table No. 1 summarizes the results for the four scenarios under consideration. Moving from "b" to "d" we move from a situation in which sophistication expectations are correct, prices are relatively close to the perfect information benchmark and sophisticated agents make important loans to *naïve* agents to a situation in which sophisticated are overoptimistic, prices react in great fashion to *naïve* expectations shocks but there is little/none trade between sophisticated and *naïve*. Case "c" falls in between these two cases. Thus there is an evident tension between the efficiency of prices and the efficiency of the consumption paths.

Table No. 1: Summary of outcomes under selected scenarios

Case	R^e	$E[w_2 \mid R^e, n^S, w_{2i}^S]$	$l(n, R^e)$	$l(E[w_2 \mid R^e, n^S, w_{2i}^S], R^e)$
A *(perfect information)*	1	1	0	0
B *(totally anticipated naïve shock)*	$\frac{1}{2}+\frac{n}{2}$	1	$\frac{2}{[1+1/n]}-\frac{1}{2}$	$\frac{1}{1+n}-\frac{1}{2}$
C *(partially anticipated naïve shock)*	$\frac{1}{4}+\frac{3n}{4}$	$\frac{1}{2}+\frac{n}{2}$	$\frac{2}{[3+1/n]}-\frac{1}{2}$	$\frac{1}{2}-\frac{2}{3+1/n}$
D *(not anticipated naïve shock)*	n	n	0	0

II.5. Some comparisons with a perfect information economy

The equilibrium of this economy deviates from what would be observed in a model with perfectly anticipated income or an economy in which there is no *naïve* price pressure and private information on fundamentals is properly aggregated. As shown in the examples of the previous subsection, the deviations involve both, the interest rate and the quantities saved individually and in the aggregate. In this subsection instead of focusing on specific outcomes we will focus on the distribution of outcomes. Complementary, numerical evaluation of expected welfare levels will be illustrated in the next section.

With perfect information the equilibrium interest rate would equal $R^* = w_2 / w_1$. In expectation the deviation between the equilibrium interest rate and the interest rate of the perfect information benchmark case equals:

$$E(R^* - R) = \frac{\bar{n} - \bar{w}}{2w_1} + \frac{(\bar{n} - \hat{n})}{2w_1\left(\frac{1}{\sigma_{\hat{n}}^2} + \frac{1}{\sigma_v^2}\right)\left(\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2}\right)}$$

Thus, the average equilibrium interest rate is different from the average benchmark interest rate due to mistaken beliefs regarding the fundamental and mistaken beliefs regarding *naïve* beliefs about fundamentals. The first term on the right side of the equation captures differences between the average beliefs of *naïve* households and the average future income. The second term captures differences between the average beliefs of *naïve* households and the average beliefs that sophisticated households hold regarding the beliefs of *naïve* investors. The importance of the first factor does not depend on the value of priors and signals precisions. This is because in this model, even under no uncertainty, sophisticated households do not offset the price pressure introduced by *naïve* households. This is a consequence of the imperfect substitutability between consumption in different periods and the restricted set of contracts that households are allowed to trade. In contrast, the second term does depend on the precision of the priors and signals; as they become more precise this term disappears.

In a similar way we can calculate the variability of the deviation of the equilibrium interest rate from the interest rate that would be observed in an economy with perfect information. The resulting expression is given by:

$$\operatorname{var}(R^*-R)=\left[\frac{\sigma_n^2\left(\frac{1}{\sigma_w^2}+\frac{1}{\sigma_e^2}+\frac{1}{\sigma_{\hat{n}}^2}\right)^2+\sigma_w^2\left(\frac{1}{\sigma_w^2}+\frac{1}{\sigma_e^2}\right)^2+\frac{1}{\sigma_v^2}}{4w_1^{\,2}\left(\frac{1}{\sigma_w^2}+\frac{1}{\sigma_e^2}\right)^2}\right]$$

To develop a simpler interpretation of this expression we assume that the average difference is zero, that is $\bar{n}=\bar{w}=\hat{n}$, in this way, the variance can be interpreted as the mean square deviation from the benchmark rate $R^*=w_2/w_1$. In addition, we assume that there are no errors in the precision attributed to the prior beliefs of n, $\sigma_{\hat{n}}^2=\sigma_n^2$. The equation above implies that the variance of the interest rate converges to $(\sigma_n^2+\sigma_w^2)/4w_1^{\,2}$ as the precision of the fundamental signal of sophisticated households converges to 1. That is, the variations in the interest rate due to mistakes in the inference from prices disappear.

Additionally, as the prior variance of *naïve* expectations or the variance of error of signal of *naïve* expectations converges to zero, the mean square deviations of the interest rate grows without bound. In other words, as the information regarding *naïve* expectations becomes precise, the interest rate reaction to information regarding *naïve* beliefs becomes so strong that, in the limit, the variance of its deviations with respect to the benchmark rate grows without bound. In particular, this means that the interest rate as an aggregator of fundamental information is a worse signal as the information on *naïve* expectations becomes more precise. This result is explained by errors in the inference process from prices that are caused by too simplistic models of others'. As indicated above, we assume that this inference process is carried out assuming that other sophisticated households receive informative signals regarding fundamentals but act naively given this data. That is, their actions are assumed to reflect more private information than they really do. This implies that market prices are assumed to be more informative that they actually are, this excessive reaction grows with the perceived precision of their information regarding *naïve* beliefs.[6] This has welfare costs for both *naïve* and sophisticated households that we will illustrate in the next section.

III. Changes in sophisticated households' skills

In this section we provide two numerical illustrations of the welfare levels associated to different levels of sophistication. First we will consider the welfare impact of an increase in the precision with which sophisticated households observe *naïve* households' expectations. Secondly, we will consider variations in the precision of sophisticated households' private signals regarding future income. In this context we evaluate the welfare impact of having sophisticated households that become increasingly better informed about fundamental aspects than *naïve* households.

In parts of the analysis we focus on the ex-ante social welfare function with equal weights on both types of households. This would be the expected welfare level of a household whose type is randomly determined according to the proportion of households of each type.

We specify a Constant Relative Risk Aversion (CRRA) utility function, in this way the distinction between decision utility and experienced utility is explicitly presented. That is, while agents' decisions satisfy a simple rule, their welfare is determined by the computation of a specific expected welfare function.

Additionally, to avoid negative consumption levels, we will impose a lower bar on second period income and consumption given by $\underline{k}$. That is, second period income is truncated at $\underline{k}$. Similarly, independently of the individual budget constraint, second period consumption is guaranteed to be at least $\underline{k}$. A borrower i will then default on their obligations if $Rl_{i}. > w_2 - \underline{k}$. In particular, if $w_2 \leq \underline{k}$ all borrowers default on their promises. For $w_2 \geq \underline{k}$ there is a fraction of borrowers that defaults on their obligations. We assume that individual default risk is shared across all lenders; we can think of a central

6. *For an early contribution on excessive volatility in asset prices see Shiller (1981).*

counterparty that implements a risk sharing scheme. In this way assuming normal distribution of w_2 and the impossibility of defaults is another simplification made during the decision process.

For a given income level $w_2 \geq \underline{k}$, *naïve* mean beliefs n and sophisticated agents' beliefs about *naïve* mean beliefs n^S, there is a critical value for a sophisticated agent private signal w_{2i}^S above which the agent will default. This critical level $\overline{w}_{2i}^S$ is defined by:

$$w_2 - \underline{k} = \frac{E[w_2 \mid R, n^S, \overline{w}_{2i}^S]}{2R} - \frac{w_1}{2}$$

The explicit expression is given by:

$$\overline{w}_{2i}^S = \sigma_e^2 \left[\frac{1}{\sigma_w^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_{\hat{w}}^2} \right] [2R(w_2 - \underline{k}) + Rw_1] - \frac{\sigma_e^2 \overline{w}}{\sigma_w^2} - \frac{\sigma_e^2 \hat{w}_{2i}^S}{\sigma_w^2}$$

Similarly, for a given income level $w_2 \geq \underline{k}$, *naïve* mean beliefs n and sophisticated agents' beliefs about *naïve* mean beliefs n^S there is a critical vale for a *naïve* agent private signal above which the agents default on their obligation. This critical level $\overline{n}_i$ is defined by:

$$w_2 - \underline{k} = \frac{\overline{n}_i}{2R} - \frac{w_1}{2}$$

which can be explicitly expressed as:

$$\overline{n}_i = 2R(w_2 - \underline{k}) + Rw_1$$

Then, the losses associated with default equal the mean value of the loans taken by agents that defaulted times the quantity of agents that defaulted. These losses generate a proportional drop in the payment that lenders receive. The loss from *naïve* agents equals:

$$L^n = Pr(n_i > \overline{n}_i) E[l(R, n_i) \mid n_i > \overline{n}_i]$$

For sophisticated agents the expression is:

$$L^S = Pr(w_{2i}^S > \overline{w}_{2i}^S)E[l(R,\hat{w}_{2i}^S,w_{2i}^S) \mid w_{2i}^S > \overline{w}_{2i}^S]$$

The adjusted return of lenders equals:

$$R' = R(L - L^S - L^n)/L$$

where L is the original total value of outstanding loans.

Throughout the next equations we assume that the parameters are given by:

$$\sigma_u^2 = \sigma_{\hat{n}}^2 = \sigma_n^2 = \sigma_w^2 = 1/10$$

We will consider variations in the precision of sophisticated households signals regarding future income (σ_e^2) and sophisticated households' signals regarding *naïve* household beliefs (σ_v^2).

Additionally we assume parameter values:

$$w_1 = \overline{w} = \overline{n} = \hat{n} = 1$$

That is, we assume that in ex-ante terms, expected future income and current income coincide and there are no systematic biases in *naïve* beliefs or sophisticated households' beliefs regarding *naïve* beliefs. Finally, we set the coefficient of relative risk aversion equal to 2.

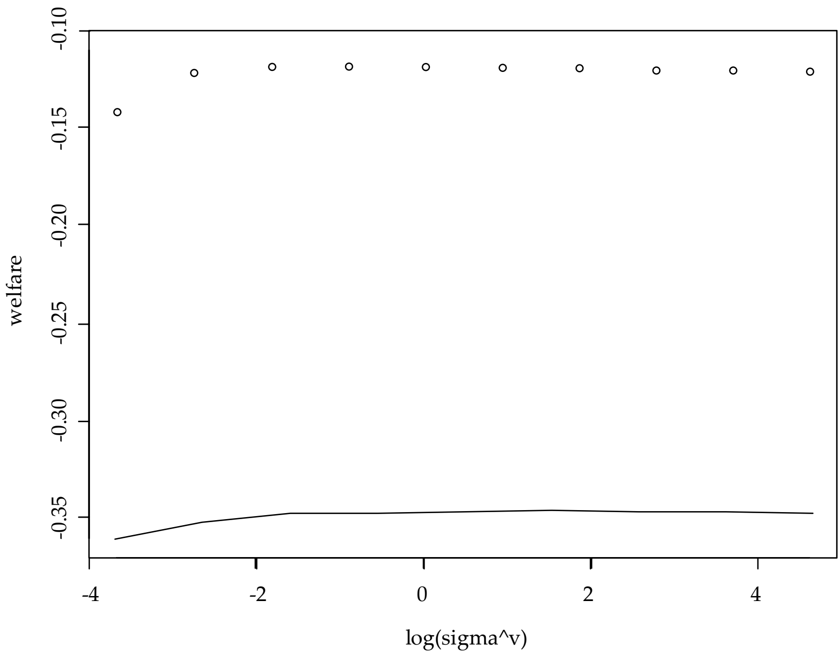
III.1. The precision of information regarding naïve households' beliefs

First, we evaluate welfare levels for different values of the precision of the sophisticated households' signals regarding *naïve*'s beliefs, that is, different values of σ_v^2. For this exercise we set σ_e^2. Figure 1 presents the approximated values of the expected value of the payoff functions of sophisticated households and *naïve* households. First, as expected, we observe that the welfare level of *naïve* households is lower than the one corresponding to sophisticated households. In addition, the values of the welfare function are relatively constant for sufficiently high values of σ_v^2. Approximately for values below $\sigma_v^2 = 1/10$ both welfare functions show a noticeable reduction in their levels as precision of the n^S signal decreases or, equivalently, as σ_v^2 increases.

As we hinted in our description of the equilibrium in the previous section, this result can be explained by the inadequate interpretation of market signals. As n^S becomes more; precise, sophisticated agents become more trustful of their interpretation of market signals. But this interpretation of market signals is based on the assumption that other sophisticated agents act more naively that what they actually do. This imperfect inference process results in excess volatility that decreases welfare levels. An example of the increment in volatility can be obtained by observing the deviations of the simulated value of the interest rate with respect to the perfect information benchmark. In our simulations the mean squared difference increases from 0,085 to 0,102 as the variance of signal n^S drops from 0,10 to 0,025.

This result is suggestive of negative externalities associated to the acquisition of information. While the present work does not deal with information choice, it can be observed that this figure indicates the existence of regions in which the choices might not incorporate negative external effects and, in this way, lead to excessive information acquisition. If the individual gains from getting information exceed the social gains, too much information could be acquired in equilibrium. It must be observed that this result occurs in a context in which there are other aspects of the learning process that are imperfect. That is, the analysis consists in evaluating the impact of changing only one of the aspects that generate incorrect beliefs. In other words, a context where the second best principle holds.

Figure No. 1: Welfare and precision of signals regarding *naïve* beliefs



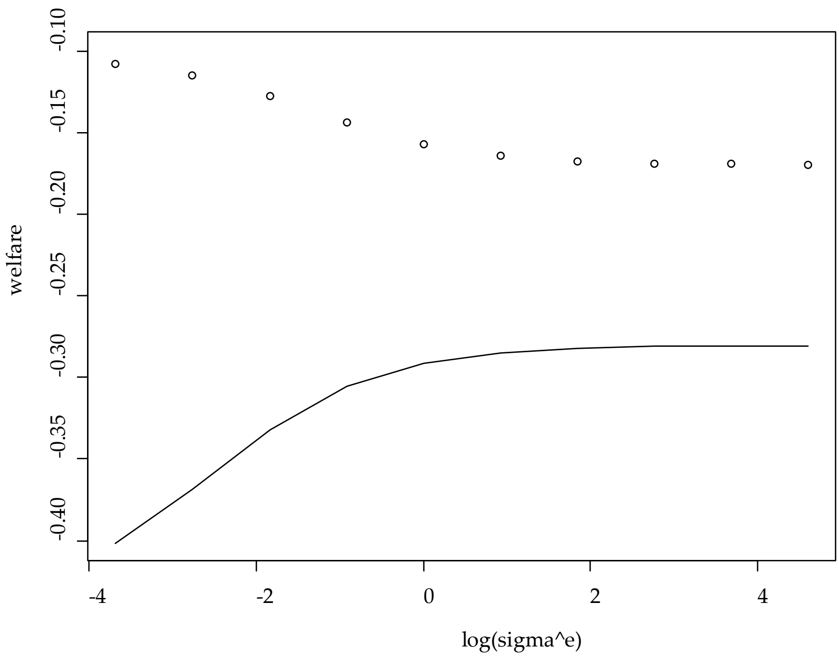
Note: Sophisticated households (dots). Naïve households (continuous line).

III.2. The precision of information regarding naïve households' beliefs

In this subsection we inspect the impact of an increment in the precision of sophisticated households' private signal regarding future income (w_{2i}^{S}). For the purpose of this analysis, we set parameter σ_v^2 equal to 0.10. Figure No. 2 presents the welfare levels calculated in the simulations. The figure shows that an increment in the precision of the signal, that is, a drop in parameter σ_e^2, is associated to an increment in the welfare level of sophisticated households and a drop in the welfare level of *naïve* households. This change is particularly noticeable for high levels of precision.

Additionally, the negative welfare impact on naive households is larger than the positive impact on sophisticated households' welfare. More precise information allows sophisticated households to trade advantageously but the private gains are smaller than the social cost, since these trades involve inefficiencies in the consumption paths.

Figure No. 2: Welfare and precision of sophisticated private signals regarding future income



Note: Sophisticated households (dots). Naïve households (continuous line).

IV. The impact of simple policy rules

In this section, an analysis of the impact of policy measures is developed. The analysis intends to illustrate the welfare impact of simple policies, in a context in which there are different levels of sophistication. We identify simple policies as those whose content is not conditioned on detailed information of the beliefs of different agents. More specifically, we will consider a rule that imposes upper and lower bounds on the value of the interest rate.

Before proceeding, it is worth keeping in mind the specific environment in which these exercises are developed. In the current simple representation, agents are trading contracts, but on aggregate terms there is an exogenously determined level of endowments to be consumed in each period. Trading does not allow aggregate intertemporal reallocations. Trading simply allows taking speculative positions based on individual beliefs.

In this context, it is to be expected that policies that restrict the ability to trade might be welfare improving. Naturally, this result might not hold in context where trading allows for aggregate intertemporal reallocation of consumption and market information can be used to make better decisions. This model can be viewed as an analysis that describes properties of an economy in a limit case.[7]

IV.1. Price collars

As indicated above, one simple policy rule might involve a constraint on the interest rate. More specifically, consider a scenario in which the only contracts that can be written involve interest rates that are inside a price collar given by $LB = 1 - r \leq R \leq UB = 1 + r$. Where r is termed collar parameter. We assume that households' observe the excess demand or excess supply and hence, make the same inferences that would be made in a market without constraints on prices. In the case of an operative constraint on R, a proportional rationing rule is assumed to resolve the excess in demand or supply.

Consider a setting in which the parameter values are as proposed in the previous section with the additional assumption that sophisticated signals precisions are as determined by $\sigma_v^2 = \sigma_e^2 = 1/10$. We evaluate welfare levels for a collar parameter ranging from 10, basically an unrestricted interest rate, to 0, constant interest rate equal to 1.

As shown in Figure Nr. 3, a price collar would significantly increase welfare level of *naïve* households without having a large negative impact on sophisticated households' welfare. This pattern indicates that if welfare is calculated in ex-ante terms, i.e. before a household learns whether it belongs to the sophisticated or *naïve* group, welfare would be increased through a restriction in price variation.

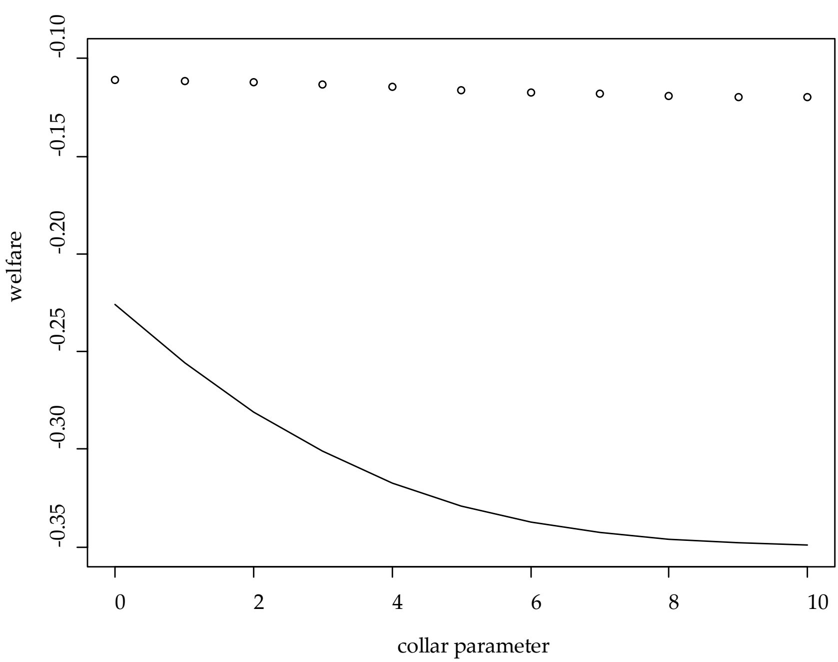
Keeping the interest rate at a constant level reduces the difference in welfare levels of sophisticated and *naïve* households but does not eliminate the gap in the welfare level for these two types of agent. This is in part because,

7. This exercise is comparable to analyses in which the performance of markets is evaluated assuming that all agents have the same abilities to form expectations and their decisions reflect well calibrated levels of confidence. The outcomes associated with this scenario should also be evaluated keeping in mind the limited range of validity.

even though prices do not change, sophisticated households benefit from trading against less sophisticated households. This result suggests that, under heterogeneous expectations, a policy that restricts price variations is not able to eliminate the inefficiencies associated to inadequate expectations.

On the other hand, sophisticated household's expected welfare is below the one they would obtain if they were able to access to a constant income level in the second period equal to the expected value of future income, that is, if they were able to eliminate uncertainty in actuarially fair terms. In this case, welfare level would be 0 which is higher than the values observed in Figure No. 3. The previous observations suggest that in this simple context, absent any intervention on beliefs, restrictions on transactions are the type policy that would do the most to restore efficiency. Naturally, in an environment in which gains from trade become sufficiently large, there will be tradeoffs associated to restrictions in trade.

Figure No. 3: Welfare with price collars



Note: Sophisticated households (dots). Naïve households (continuous line).

V. Discussion

The present work constitutes a simple exercise that explores market outcomes associated to heterogeneity in the ability to form expectations and learn from other households' decisions. The exercise suggests that sophisticated agents' conduct is not enough to guarantee an efficient allocation. In addition, increases in certain learning skills might be associated to negative externalities and lower welfare levels. Finally, restrictions in price variation might increase welfare in ex-ante terms but need not restore efficiency.

In this way, the present study advances the understanding of environments characterized by heterogeneity in abilities to form expectations. The results provide a sharp contrast with what would be observed under unbounded or homogeneous abilities. The evaluations developed in the present work could be analyzed in more general contexts or under more general profiles of learning processes. In the following paragraphs, we provide a brief discussion regarding robustness of our findings and directions for future research.

V.1. On profiles of skills

We have focused on the detailed analysis of the outcomes associated with alternative profiles of sophistication in a simple environment. We made a strong separation between sophisticated and *naïve* households in order to carry out a more tractable analysis. We believe that our qualitative results would not change if we allow for a more gradual differentiation between households with different levels of skills.

We assume that each household has certain level of correct information about the environment, some information is absent and sometimes they are aware of which information they possess and which information they do not possess. Our model does not develop a dynamic model of how this profile of skills and information emerges. In particular, our contribution does not suggest that the given profile of skills is in a stationary state and it does not suggest a process through which that profile of skills is attained.

A framework that explains this type of profiles would bear in mind that different agents make use of cognitive processes at different frequencies and with different intensities. Additionally, in some circumstances, behavior is be governed mostly by some acquired habits, easily available information, rules of thumb to assess future income or simple intuitive signals. These processes have low cognitive costs, might occur below awareness and are typically triggered automatically. Despite its low cognitive costs these processes can generate very suitable decisions as in the case of well trained intuitive responses. Behavior is also be shaped by more sophisticated processes such as the consideration of hard to get information, the evaluation of hypothetical scenarios and inferences through logical rules.

The use of alternative cognitive processes and the development of skills that improve their output are a function of history and the perception of the individual stakes associated to the decision problem. It is not necessarily the case that learning process should converge to a state in which all players end up with high sophistication levels. Tversky *et al.* (1974) suggest that biases in intuitive judgments might not be eliminated if the errors in past predictions are not properly processed. Even for the case of sophisticated households, it is not necessarily the case that their models of others will converge to the right model. For convergence to occur environment needs to be stable, the feedback needs to be frequent and results have to be processed in a way that the mistake is identified.

V.2. On alternative environments

The environment in which we carried out the analysis is highly stylized. This choice was made with the intention of facilitating the identification of the channels through which a rich profile of learning processes affects outcomes. Nevertheless, for the analysis of some relevant economic phenomena the environment considered in this model needs to incorporate alternative features. In this section we give examples of which features appear as natural candidates.

One characteristic of our exercise is the property that actions are strategic substitutes. That is, abstracting from informational inferences, higher demand for loans by a fraction of the households increases the market clearing prices and, thus, leads to lower demand levels by the remaining households. Richer phenomena would be observed if strategic complementarities in actions were allowed. Complementarities in actions are likely to occur in an economy with production and investment.[8] Additionally, complementarities in actions can generate complementarities in information acquisition. That is, if more agents are responding to a set of information the incentives to acquire that information become stronger (see Veldkamp, 2011).

Another complementary aspect that were not considered is the possibility of bankruptcy and the ability to trade different types of financial contracts. The type of contract considered simply involves the exchange of current units of a single good for future units of the unique good on that date.

A more complex contractual setting opens the question about the capacity of agents to anticipate the distribution of payoffs associated to a transaction. As in the case of anticipating future income, it is reasonable to assume that, in a complex contractual setting, agents' capacity to understand the implications of alternative contracts is asymmetric. As in the case of strategic complementarities we believe that contractual issues can bring interesting insights about how different profiles of sophistications can be associated to alternative economic outcomes.

8. See Cooper et al. (1988) for a general perspective on the impact of strategic complementarities in macroeconomics. See Haltiwanger et al. (1985; and 1989) for an analysis of naïve behavior in an environment with complementarities.

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