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SPATIAL COMPETITION AND THE LOCATION OF FIRMS WITH NON-UNIFORMLY DISTRIBUTED COSTUMERS

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ABSTRACT

This paper develops a free-entry spatial model that provides theoretical assessments for explaining firms' prices and locations with a non-uniform costumers' distribution. Costumers, located on the real line, incur in a cost proportional to the distance to the firm for buying its goods. This model accounts for firms' tendency to concentrate in high density regions and to charge lower prices. A solution algorithm is presented and comparative static exercises are used to show the effect of changes in the transportation costs and dispersion of costumers.

Keywords: location; spatial models.

RESUMEN

Este trabajo desarrolla un modelo de libre entrada espacial que provee los lineamientos teóricos para explicar los precios y las ubicaciones de las firmas con clientes que están distribuidos en forma no uniforme. Los clientes, localizados en la línea de números reales, incurren en un costo proporcional a la distancia a la firma para comprar sus bienes. El modelo explica la tendencia de las firmas a concentrarse en regiones de alta densidad y cobrar precios bajos. Se presenta una solución al algoritmo de ubicación y ejercicios de estática comparada para mostrar el efecto de cambios en los costos de transporte y de la dispersión de los clientes.

Palabras Clave: ubicación, modelos espaciales.

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I. Introduction

Spatial models are extensively used in economics to model differences in the firm's market power and product differentiation. Their use exceeds the original "geographical" interpretation to accommodate any dimension that differentiates firms in a given market. Studies on the firms' location (firms optimally decide how to accommodate to consumer preferences for maximizing profits and the strategic interaction thereof) were first developed along the lines of the simple Hotelling (1929) linear city and Salop (1979) circular city style models; the former mostly interested in a two-firm location game along the unit interval, and the latter on the number of entries and their optimal separation in the unit circle (see Tirole, 1988, ch.7, for a review).

There are many empirical applications that justify further theoretical work on this subject. Lewis (2008) finds that the extent of price dispersion in retail gasoline sellers is related to the density of local competition; Syverson (2007) shows lower factor prices and more efficient firms in denser regions for ready-mix concrete plants; Dranove, Gron and Mazzeo (2003) study differentiation and competition in Health Maintenance Organizations (HMO) local markets. One particular case where this type of models has attracted interest is the banking industry, where it has been argued that concentration and market size explain much of the inter-regional differences in prices of banking services. For instance, Pilloff and Rhoades (2002) provide empirical evidence on the effect of the market size and concentration measures on banks' profitability. They argue that larger market sizes are associated with lower profit rates, because higher profits are a magnet for entry and thus are unsustainable. They also show that income per capita (or wealth) is a possible indicator of market's attractiveness. Moreover, Petersen and Rajan (2002) show that the distance to the nearest bank is a good predictor of the cost that small firms face for obtaining credit and Felici and Pagnini (2008) study the entry of banks in Italy and the effect of distance. Montes-Rojas (2008) uses studies the interconnection between geographical competition and the banking sector using a simpler model to the one used in this paper.

A non-uniform customers' distribution introduces additional features to these models and complicates the characterization of the optimal location of firms. For that reason, only a few studies work with non-uniform distributions. Neven (1986) uses a two-firm linear city model to study a location-price game, which is suitable to study the features of the density function that produce different degrees of differentiation: a higher concentration of wealth produces closer locations. Tabuchi and Thisse (1995) use a triangular and symmetric distribution. In this case, no symmetric equilibrium exists and those authors show the existence of asymmetric equilibriums, characterized by strong product differentiation. Anderson, Goeree and Ramer (1997) provide a general model for studying the effect of interaction on the firms' location.

We contribute to this literature by constructing a general theoretical model that allows studying the optimal location of an infinite sequence of firms under general non-uniform costumers' distributions. In contrast to other studies, our model considers an infinite sequence of firms located over the entire real line. We show existence of Subgame Perfect Nash Equilibria under very general conditions on the distribution of costumers. In our model, the interaction properties among firms are similar to those in the circular city and we focus on the pattern of firms' locations rather than on the number of entries (which is infinite). The model shows that the firms' distribution varies according to the customers' distribution and generates intuitive location, price and profit patterns. The intuition behind the model can be summarized as follows: in regions with high density of potential costumers firms are compelled to become closer to avoid potential entrances, which produce more competition, and consequently lower prices.

We use comparative static exercises to study changes in transportation costs and distributional features (dispersion). The model shows that lower transportation costs produce more distance between contiguous firms but lower prices. Moreover, less dispersion in the distribution of costumers produces lower average distance between contiguous firms and lower prices when we consider regions close to the mode of the distribution. However, this pattern does not apply when we consider regions distant from the mode.

The paper is structured as follows. Section II describes the model. Section III develops the equilibrium concept. Section IV presents an algorithm to solve the model and comparative static exercises. Section V concludes. Mathematical proofs are condensed in the Appendix.

II. Costumers and firms

Consider a modification of the linear city model (Hotelling, 1929). A market is represented by the real line and has potential costumers (we will use consumers, costumers and individuals interchangeably) located in "regions" denoted by the continuous variable $x \in \Re$. The population in each region is given by the measurable function $x \rightarrow f(x)$. If the distribution of costumers across regions is uniform, $f(x)$ will be a constant (without loss of generality 1). Otherwise consider the following assumption.

Assumption 1 (Non-uniform distribution of costumers): $f : \Re \rightarrow \Re$ is a measurable function which is twice continuously differentiable and satisfies

(i) $f(x)$ has a unique maximum at $x=0$,
(ii) $f(x)=f(-x)$ (symmetry),
(iii) $f(.) \geq 1, f'(x) \geq 0$ for $x<0$, $f'(x) \leq 0$ for $x>0$, $f'(0)=0$,
(iv) $\int (f(x) - 1) \, dx = 1$,
(v) $\lim_{|x| \to \infty} f(x) = 1$, $\lim_{|x| \to \infty} f'(x) = 0$.

Condition (iii) implies that $\int f(x) \, dx = \infty$, but condition (iv) states that $f(.) - 1$ is a proper density function. x=0 represents the largest region in the market, and costumers' density decreases as $|x|$ becomes larger. Moreover, as $|x| \rightarrow \infty$ the population becomes uniformly distributed and $f(x) \rightarrow 1$. A simple way to generate this measure is to consider a uniform measure (i.e. $u(x)$=1 for all x) and to add a symmetric density function (i.e. standard normal).

Assume that customers may consume their wealth (which provide a utility of 0) or buy firms' goods that are valued at v>0. In that case, individuals pay a transportation cost proportional to the distance to the nearest firm, denoted by h. All individuals have a wealth of 1 unit and are identical except for their location. Their utility is then $max\{v\text{-}p\text{-}hd,0\}$, where p is the price paid for the firm's good and d is the distance to the nearest firm.

Consider also an infinite sequences of firms, indexed by i=0,1,2,…, that play a two stages non-cooperative game. In the first stage, firms i=0,1,2,… simultaneously decide whether or not to enter the market (let $\{E_i\}_{i=0}^{\infty}$ be the entry decisions), and if they do, they choose a location $\{z_i\}_{i=0}^{\infty}$, where z_i denotes location on the real line. The game proposed here is a modification of the Economides' (1989) model. This author used a three stage game to compute symmetric equilibrium in a circular city model, where the first stage is entry, the second location and the third price competition. We simplify the exposition by compressing his first two stages. See also Eaton (1976) for a similar model in an unbounded space.

In the second stage they compete on prices. Firms can charge two different prices (one to the left and one to the right), denoted by $\{P_i^-, P_i^+\}_{i=0}^{\infty}$. Firms have a unique location and face an entry cost given by 0< s <∞ and a constant cost per good sold 0< c <∞. The firms' profits are given by equation (6) below.

Two key features of the model need to be discussed. First, the use of an infinite measure of costumers on the real line is made for technical reasons and because the main purpose of this model is to study differences across regions (high vs. low density) rather than the number of firms entering the market. The most difficult issue in modelling the location of firms is the existence of a border. For instance, many papers consider a [0,1] interval for the location of firms (as in the Hotelling, 1929, model). The main problem is that modelling equilibrium is difficult because firms may prefer to move towards the extremes to capture additional market. An alternative approach is to impose a circular city model although this does not guarantee the existence of equilibrium (see D'Asprenmont, Gabszewicz and Thisse 1979, Gabszewicz and Thisse 1986 and Tirole 1988, ch. 7, for a general discussion). In contrast considering an infinite measure has the main advantage that we can construct equilibria where we do not have to consider the strategic effects on the border.

Note that, for any density function on the real line, the density should go to zero as $|x|\to\infty$. This implies that there is a firm located on an extreme (i.e. no other firm will locate to its right or left). In practice, this means that we will have the same problems as if we would have a border, and consequently equilibrium may not exist. The reason is that as the mass goes to zero, no additional firms may find profitable to locate further in the extremes. Because of this "extreme effect", the mass of costumers never goes to zero so that the "entire real line" will be served (see Assumption 1). An additional restriction on v need to be imposed in order to guarantee that the entire real line is served. In both Lemma 2 and Lemma 3 below we have $v \geq \sqrt{s/2h}$. This condition guarantees that firms will locate in the uniform section of the distribution of costumers, and therefore, the entire real line will be served.

Second, the bidirectional (left and right) price discrimination structure is another simplifying assumption made for technical reasons. By imposing this structure we only need to solve pair-wise Nash equilibria in the second stage (price competition). If the distribution of costumers were uniform, this assumption would make no difference as firms will behave in a symmetric fashion. In our case of a non-uniform distribution, this happens as $|x| \to \infty$. However, in the non-uniform region imposing a unique price in both directions may not produce uniqueness of the Nash equilibria because firms may be indifferent between charging a high price to concentrate on costumers located towards the high density region or a low price to attract costumers towards the low density region.

Nevertheless, a heuristic argument can be made to make this assumption less critical. The purpose of the model is to study firms' patterns across regions. If we consider a sufficiently smooth measure $f(.)$ and consider a firm whose clients are in a small interval about its location (i.e. $x_i^+ - x_i^- < \varepsilon$, with $\varepsilon > 0$ small, as defined in section III.A.), then the difference between the right and left prices will be small for negligible differences in the costumers' measure (i.e. $|f(x_i^+) - f(x_i^-)|$ small). However, when comparing across different regions, differences in prices and locations will still appear and then, both right and left prices will have similar inter-regional patterns as they depend on the average density about the firm's location. Additional future research would be needed to relax this assumption and to investigate whether or not this matter for the conclusions of this model.

III. The equilibrium

The equilibrium concept is Subgame Perfect Nash Equilibrium (SPNE). For fixed locations, Nash equilibrium can be easily found in the second stage by computing the prices strategies as in a standard Hotelling model. The first stage corresponds to entry and location. The location sequence may not be unique because the entire real line is to be *filled*: Lemma 2 below shows two SPNE location patterns in the uniform case which generates either zero or positive profits (for a discussion about the multiplicity of equilibriums see Eaton, 1976). A formal definition of the equilibrium concept is:

Equilibrium: A SPNE of the spatial game is given by a sequence $\{E_i^*, z_i^*, P_i^{-*}, P_i^{+*}\}_{i=0}^{\infty}$ such that for each firm i=0,1,2…, $\{P_i^{-*}, P_i^{+*}\}_{i=0}^{\infty}$ maximizes the firms' profits given in (6) (and they are non-negative) for fixed locations; and $\{E_i^*, z_i^*\}_{i=0}^{\infty}$ is also a Nash equilibrium of the first stage subgame taking into consideration the terminal node price strategies (a Nash equilibrium here means that each i firm has no incentives to deviate given the strategies of the i=0,1,…i-1,i+1… other firms).

III.1. Second stage: price competition

Through this subsection we assume that locations are fixed. Assume that a firm is located in $z \in \Re$ of the line-market. It will attract individuals from the left (-) and from the right (+). The scope of each firm is given by those individuals who prefer buying firm services at the z location.

Let $\{P_i^-, P_i^+\}$ be the prices that the i^{th} firm, which is in $z_i \in \Re$, charges to its left and right respectively. P_{+i}^- is the (left) price of the nearest firm from the right, which is in $z_{+i} \in \Re$ ($z_{+i} > z_i$). P_{-i}^+ and z_{-i} ($z_i > z_{-i}$) are the (right) price and location of the nearest firm from the left. Moreover, let x_i^+ be the distance between firm i (at z_i) and the consumer who is indifferent between buying services in z_i and z_{+i}. x_i^- is the distance between firm i and the individual who is indifferent between buying services in z_i and z_{-i}.

The *indifferent* individuals can be found using the following indifference equations:

$$P_i^+ + hx = P_{+i}^- + h\left[(z_{+i} - z_i) - x\right] \qquad , \qquad (1)$$

$$P_i^- + hx = P_{-i}^+ + h\left[(z_i - z_{-i}) - x\right] \quad . \tag{2}$$

The firm's scope for both sides can be easily obtained as:

$$x_i^+ = \frac{P_{+i}^- - P_i^+ + h(z_{+i} - z_i)}{2h} \quad , \tag{3}$$

$$x_i^- = \frac{P_{-i}^+ - P_i^- + h(z_i - z_{-i})}{2h} \quad . \tag{4}$$

Adding both equations,

$$X_i = x_i^- + x_i^+ = \frac{1}{2h}\left(P_{+i}^- + P_{-i}^+ - P_i^- - P_i^+ + h(z_{+i} - z_{-i}\right). \tag{5}$$

The scope of each firm depends on its price and location, the prices and locations of its rivals and the costs of entry, production and transportation. Define $Q_i^- = \int_{z_i - x_i^-}^{z_i} f(x)dx$ and $Q_i^+ = \int_{z_i}^{z_i + x_i^+} f(x)dx$ as the demand served for firm i. Each firm i maximizes the following profit function:

$$\begin{aligned}\Pi_i &= (P_i^- - c)\int_{z_i - x_i^-}^{z_i} f(x)dx + (P_i^+ - c)\int_{z_i}^{z_i + x_i^+} f(x)dx - s, \\ &\equiv (P_i^- - c)Q_i^- + (P_i^+ - c)Q_i^+ - s\end{aligned} \tag{6}$$

subject to $P_i^-, P_i^+, x_i^-, x_i^+ \geq 0$. If we assume that the prices and places of all the firms are given, the i^{th} firm will be a local monopolist.

Assuming an interior solution and using the Leibniz rule, the first order condition is:

$$\begin{aligned} Q_i^+ - \frac{1}{2h}(P_i^+ - c)f(z_i + x_i^+) = 0, \\ Q_i^- - \frac{1}{2h}(P_i^- - c)f(z_i - x_i^-) = 0. \end{aligned} \tag{FOC}$$

The second order condition requires that:

$$-\left(\frac{1}{h}f(z_i - x_i^-) + \frac{P_i^- - c}{4h^2}f'(z_i - x_i^-)\right) \leq 0$$

and

$$-\left(\frac{1}{h} f(z_i + x_i^+) - \frac{P_i^+ - c}{4h^2} f'(z_i + x_i^+)\right) \leq 0$$.[1]

The firm responds in the usual way to an increment in the distance or price of the rival firms. The following lemma shows the existence and uniqueness of Nash equilibrium.

Lemma 1: *Consider Assumption 1. For fixed locations, there exists a unique Nash equilibrium in the second stage subgame.*

Proof: In the Appendix.

III.2. First stage: location

Define Z, the space of spatial locations, as the set of all possible collection of locations of the form $\{z_j\}_{-\infty}^{\infty}$ with $-\infty < z_{j-1} < z_j < z_{j+1} < \infty$ for all j. Let $\{j\}_{-\infty}^{\infty}$ denote a spatial ordering generated by those locations. Note that for any sequence of firms' locations there is only one spatial location. Hereafter the index j refers to a spatial location sequence and i to a firm location sequence. Since for fixed locations prices are unique, any SPNE can be seen as an element of Z with an induced price sequence. Define $\Pi\ (z;\underline{z},\bar{z})$ as the profit obtained (as in (6)) by a firm located at $z \in (\underline{z},\bar{z})$ with its closest competitors at $(\underline{z},\bar{z})$. Also define $\Delta(\underline{z},\bar{z}) = \arg\sup_z \Pi(z;\underline{z},\bar{z})$.

Consider the following sequences that characterize location patterns:

$\{z_j\}_{-\infty}^{\infty}$ is a *profitable entry-deterrence* sequence if it is a spatial location such that $\Pi(z_j;z_{j-1},z_{j+1}) \geq 0$ and $\Pi(\Delta(z_j,z_{j+1});z_j,z_{j+1}) \leq 0$ for all j.

$\{z_j\}_{-\infty}^{\infty}$ is a *maximum distance* sequence if it is a spatial location such that $\Pi(z_j;z_{j-1},z_{j+1}) \geq 0$ and $\Pi(\Delta(z_j,z_{j+1});z_j,z_{j+1}) = 0$ for all j.

$\{z_j\}_{-\infty}^{\infty}$ is a *zero-profit* sequence if it is a spatial location such that $\Pi(z_j;z_{j-1},z_{j+1}) = 0$ and $\Pi(\Delta(z_j,z_{j+1});z_j,z_{j+1}) \leq 0$ for all j.

1. Each condition corresponds to the difference of two terms. The first term is the additional gain in demand from reducing the price; the second is the loss in profits (i.e. P-c) extrapolated at the derivative of f(.). This is satisfied for the uniform measure but it needs to be checked for non-uniform measures.

A *profitable entry-deterrence* sequence does not have any additional profitable entry. Also note that *maximum distance* and *zero-profit* sequences are special cases of the former.

Lemma 2: *Consider a uniform distribution of costumers (i.e. $f(x)=1$ for all x). Assume that* $v \geq \sqrt{s/2h}$ *. Let* $\{j\}_{-\infty}^{\infty}$ *denote a spatial ordering. Then any symmetric (i.e.* $z_j^* - z_{j-1}^* = z_{j+1}^* - z_j^*$*) profitable entry-deterrence sequence is a SPNE. Consequently a zero-profit SPNE exists and has a location sequence satisfying* $z_j^* - z_{j+1}^* = \sqrt{s/h}$ *for all j, and a maximum distance SPNE exists and has a location sequence* $z_j^* - z_{j+1}^* = \sqrt{2s/h}$ *for all j.*

Proof: In the Appendix.

Unlike the uniform case, in a non-uniform distribution for fixed locations of their competitors, firms may prefer to move towards higher density regions. Condition (7) below implies that when considering the remaining firms' location as fixed, no deviation, which may attract a new entrance, would produce a profit higher than the one obtained at equilibrium.

Let $\{z_j^*\}_{-\infty}^{\infty}$ be a *location compatible* sequence if it is a spatial location such that for all *j*, $\Pi(z_j^*; z_{j-1}^*, z_{j+1}^*) \geq 0$, $\Pi(\Delta(z_j^*, z_{j+1}^*); z_j^*, z_{j+1}^*) = 0$ and

$$\max\left\{\sup_z \Pi\left(z; z_{j-1}^*, \Delta(z, z_{j+1}^*)\right), \sup_z \Pi\left(z; \Delta(z_{j-1}^*, z), z_{j+1}^*,\right)\right\} \leq \Pi_j^*. (7)$$

This condition is satisfied in the uniform distribution equilibrium described in Lemma 2. With a non-uniform distribution of costumers, if a firm deviates from its original location (i.e. moving towards the high density regions), it can do so without attracting new entrances. Moreover maximum distance is not enough to ensure the existence of SPNE. The following Lemma provides a sufficient condition to get a SPNE.

Lemma 3: *Consider a non-uniform distribution (as in Assumption 1). Assume that* $v \geq \sqrt{s/2h}$*.Then, any location compatible sequence is SPNE.*

Proof: In the Appendix.

Note that as $|x| \to \infty$ and $f(x) \to 1$ the location sequence becomes that described in Lemma 2. We will use the term "convergence" for a location sequence that becomes that of the uniform case as $|x|\rightarrow\infty$. Note that if the location sequence converges, location patterns, prices and profits will also converge to the uniform case. We will say that a sequence converges *faster* than other sequence, if for a given discrepancy ε in terms of location patterns, prices or profits, a smaller value of $|x|$ is required to observe convergence to the uniform case.

IV. A solution algorithm and comparative static exercises

Unfortunately, in general the model does not provide an explicit analytical solution for non-uniform distributions of customers. The reason is that solving the FOCs requires solving a system of two non-linear equations with two unknowns and it involves using both the density and the distribution functions (see Lemma 1 for a discussion and existence of solution).

To study the distribution pattern of firms across regions we use a normal non-uniform distribution of costumers, $f(x) = \phi(x;\sigma) + 1$ where:

$$\phi(x;\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

is a normal density function with mean zero and standard deviation σ.[2] It can be easily checked that $f(x)$ satisfies Assumption 1, that is, it is symmetric about zero and converges to 1 as x goes to infinity.

In this subsection it is showed that in equilibrium firms are more concentrated in high density regions and more disperse in low density ones. Therefore, high density regions will face lower prices. Moreover, the average transportation cost in a low density region will be higher than the one in a high density region. Comparative static results where we consider changes in the transportation costs (h) and the dispersion of costumers (σ) are provided in the following subsections.

To show how the model works, consider a simulation with parameters c=1, s=1.25, h=10, σ =1.[3] In equilibrium, as x goes to infinity and the costum-

2. *Similar results were obtained for other unimodal distributions.*

3. *A high value of h is used to have a significant location sequence in the non-uniform portion of the distribution. Similar results may be achieved by expanding the density mass (i.e. multiplying by a constant).*

ers' distribution becomes uniform we will obtain equal spacing of firms, with:

$$d_U = \sqrt{\frac{2s}{h}} = 0.5 \;;$$

profits will be $\Pi_U = s = 1.25$ and prices $P_U = 2s / d + c = 6$ (see Lemma 2). In other words, the distance between contiguous firms, prices and profit sequences will converge to the uniform case. The next paragraph describes the simulation exercise.

Since the distribution of wealth is unimodal, assume that a firm is located at the mode. This requirement is not necessary (the equilibrium is not unique) but it simplifies the computation. Let this firm be $i = 0$ (then $z_0 = 0$). Moreover, let the firms located in the positive locations be odd, and in the negative be even. Therefore $z_1 \in (0,\infty)$, $z_2 \in (-\infty,0)$, $z_i > z_1$ for i odd and $z_2 > z_i$ for i even. Consider first the location of the $i = 1$ firm. Entry deterrence is achieved by any location $0 < z_1 \leq z_1^*$, where:

$$z_1^* = \sup_z \left\{ z > 0 : \sup_x \Pi(x;0,z) \leq 0 \right\}$$

To achieve a *location compatible sequence* we set $z_1 = z_1^*$. To compute z_1^* we use the following algorithm:

- Construct a grid spacing of 0.005, i.e. $\Omega_1 = \{0, 0.005, 0.01, 0.015, \ldots\}$.
- For each $z \in \Omega_1$, construct $x(z) = \left\{ x \in \Omega_1 : \max_x \Pi(x;0,z) \right\}$.
- Define $z_1^* = \max_z \left\{ z \in \Omega_1 : \Pi(x(z);0,z) \leq 0 \right\}$

The same process can be applied to i=3, by choosing

$$z_3^* = \sup_z \left\{ z > z_1^* : \sup_x \Pi\left(x; z_1^*, z\right) \leq 0 \right\}$$

and to higher odd-numbered firms. In that case we use the following algorithm, for $i = 3,5,7,\ldots$

- Construct a grid spacing of 0.005, i.e.
  $\Omega_i = \{z^*_{i-2}, z^*_{i-2} + 0.005, z^*_{i-2} + 2\times 0.005, z^*_{i-2} + 3\times 0.005, \ldots\}$.
- For each $z \in \Omega_i$, construct $x(z) = \left\{ x \in \Omega_i : \max_x \Pi\left(x; z^*_{i-2}, z\right)\right\}$.
- Define $z^*_i = \max_z \left\{ z \in \Omega_i : \Pi\left(x(z); z^*_{i-2}, z\right) \leq 0 \right\}$

Finally, by symmetry, set $z^*_i = -z^*_{i+1}$ for $i = 1,3,5\ldots$ This algorithm is programmed in MATLAB and it is available from the author upon request.

Figure Nr 1*a* shows the computed location and distance between contiguous firms, that is, the graph $\{(z^*_i, z^*_{i+2} - z^*_i) : i = 0,1,3,5....\}$. It can be noted that firms become more disperse as the distribution of costumers becomes more uniform. Therefore, high density regions will face lower average transportation costs. Figure Nr. 1*b* plots the price sequences $\{(z^*_i, P^{-*}_i), (z^*_i, P^{+*}_i) : \mathrm{i}=0,1,3,5...\}$. Prices increase monotonically as x increases. This produces that high density regions would also face lower prices. The intuition behind the model can be summarized as follows: in regions with high density of potential costumers, firms are compelled to become closer to avoid potential entrances, but this results in more competition, and consequently lower prices. Also, note that there exist small differences between P^{-*} and P^{+*}: for i=0 both prices are equal, and we have that $P^{-*}_i \leq P^{+*}_i$, i=0,1,3,5.... The reason is that for x>0 (i.e. i odd), P^- represents the price charged towards higher density regions, and by lowering the price, the firm would be able to attract more individuals. At the same time, each firm faces more competition towards the higher density direction. Finally Figure Nr. 1*c* reports the profit sequence $\{(z^*_i, \Pi^*_i) : i = 0,1,3,5....\}$. In this case, no clear pattern emerges: if on the one hand, more competition translates into lower prices (and lower profits); on the other hand more density produces larger profits.

Figure Nr. 1*a*. Location and distance between contiguous firms

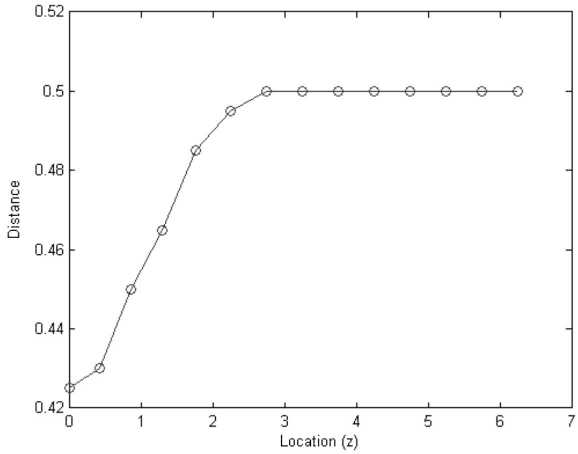


Figure Nr. 1*b*. Location and prices

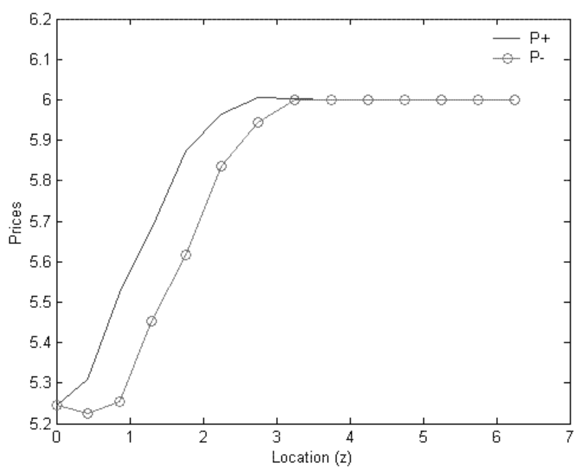
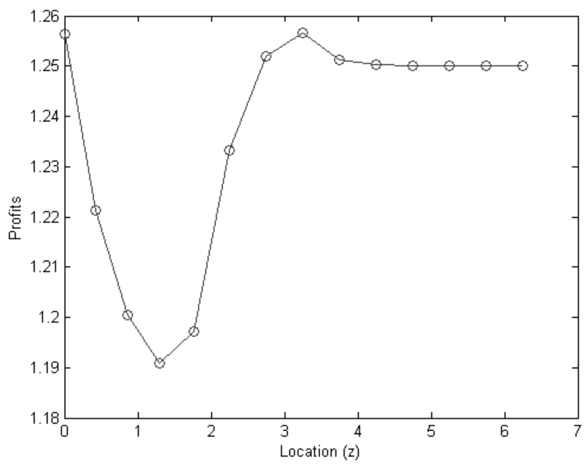


Figure Nr. 1*c*. Location and profits



The model provides a simple framework to study the effect of the introduction of a new technology (a reduction in h, the transportation cost). As Lemma 2 shows, in equilibrium with a uniform distribution, the distance between contiguous firms is proportional to $\sqrt{s/h}$. Moreover prices would be proportional to h, that is

$$P(h) = c + hd(h) = c + A\sqrt{sh}\,,$$

where A is a constant that depends on the SPNE considered but not on h. Consequently, a reduction in h would produce higher dispersion among firms but lower prices. *A priori*, we expect a similar effect to appear in the non-uniform case. For the non-uniform case, we provide simulations to study the effect of a change in h. We construct simulations where h takes values in {6,8,10}. Figures Nr. 2*a-c* reports the graphs

$$\left\{\left(z_i^*(h), z_{i+2}^*(h) - z_i^*(h)\right) : i = 0,1,3,5....,h = 6,8,10\right\},$$

$$\left\{\left(z_i^*(h), P_i^{-*}(h)\right), \left(z_i^*(h), P_i^{+*}(h)\right) : i = 0,1,3,5...,h = 6,8,10\right\},$$

$$\left\{\left(z_i^*(h), \Pi_i^*(h)\right) : i = 0,1,3,5....,h = 6,8,10\right\}.$$

Figure Nr. 2*a*. Location and distance: Different transportation costs

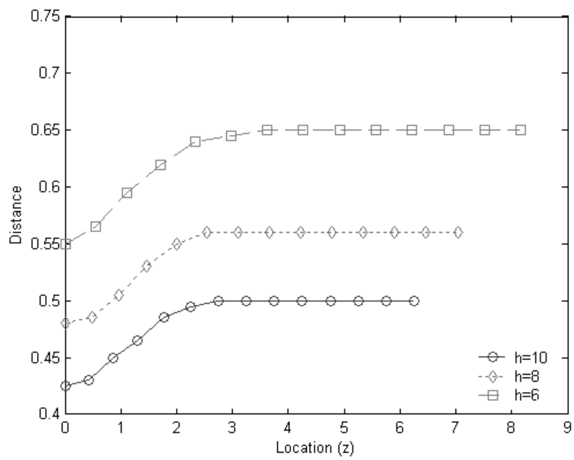


Figure Nr. 2*b*. Location and prices: Different transportation costs

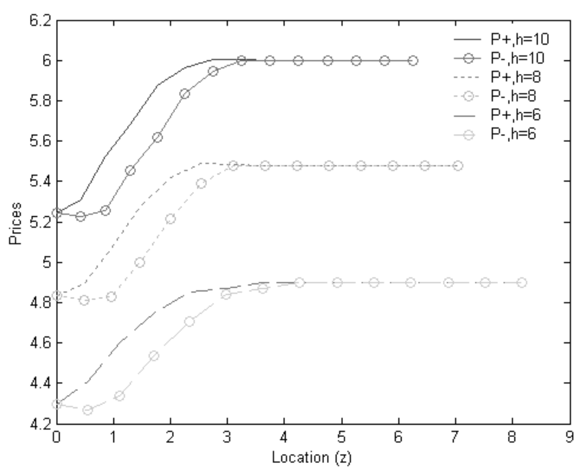
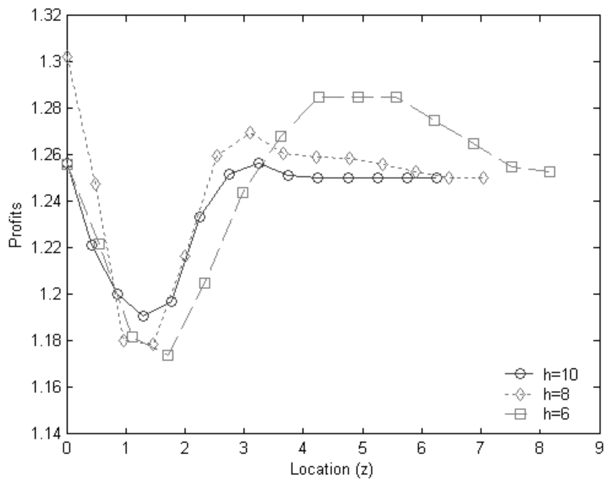


Figure Nr. 2*c*. – Location and profits: Different transportation costs



The simulations show similar results to those obtained in the uniform case, that is, as *h* decreases (i.e. better transportation technology), the distance between contiguous firms increases, prices decrease and there is no clear effect on profits.

As a final exercise we study the effect of different concentration of costumers, as measured by different values of the dispersion parameter. We construct simulations where σ takes values in {1,2,3}. Figures Nr. 3*a*-*c* reports the graphs

$$\left\{\left(z_i^*(\sigma), z_{i+2}^*(\sigma) - z_i^*(\sigma)\right) : i = 0,1,3,5....,\sigma = 1,2,3\right\},$$

$$\left\{\left(z_i^*(\sigma), P_i^{-*}(\sigma)\right),\left(z_i^*(\sigma), P_i^{+*}(\sigma)\right) : \text{i}=0,1,3,5...,\sigma=1,2,3\right\},$$

$$\left\{\left(z_i^*(\sigma), \Pi_i^*(\sigma)\right) : i = 0,1,3,5....,\sigma = 1,2,3\right\}.$$

Figure Nr. 3*a*. Location and distance: Different concentration of wealth

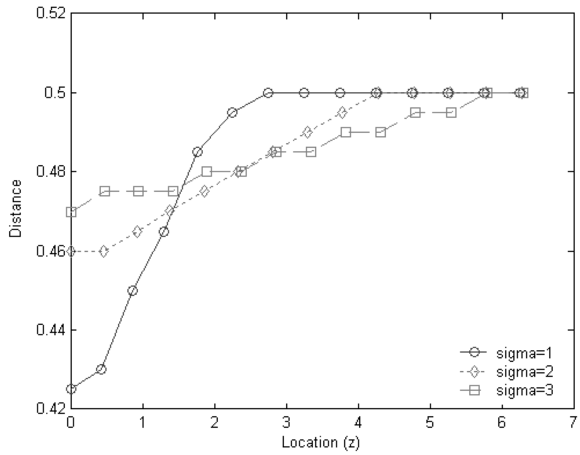


Figure Nr. 3b. Location and prices: Different concentration of wealth

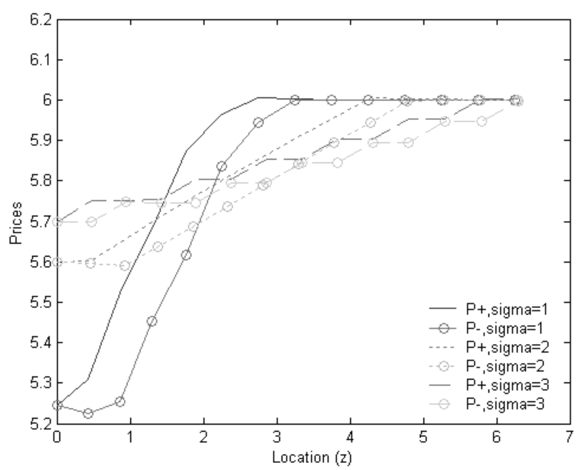
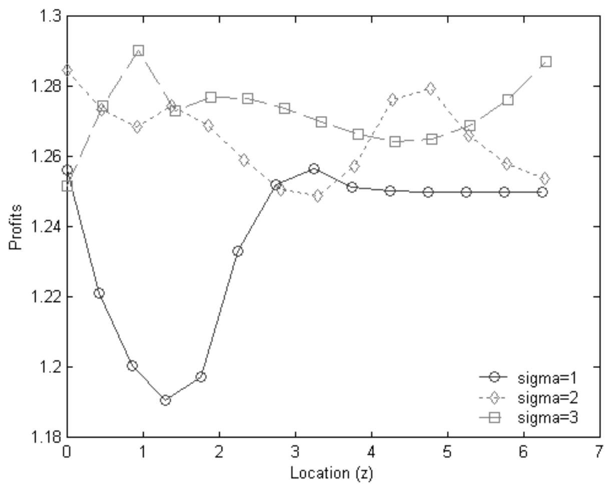


Figure Nr. 3c. Location and profits: Different concentration of wealth



We observe that the smaller the value of dispersion, the faster the convergence to the uniform case distance between contiguous firms (=0.5) and prices (=6). Regions near x=0 would face lower average distance and prices for a smaller value of σ. However individuals located before the convergence to the uniform case but not close enough to x=0, prefer higher values of σ. The same pattern is observed for profits, that is, faster convergence for smaller values of σ. In this case, for σ =3, the range considered is not enough to observe convergence.

Although not reported, we also explore changes in c and s. The former being the marginal cost has a direct effect on prices. A reduction in c decreases prices and produces higher dispersion among firms. The latter being the fixed cost has an effect on the market power that each firm faces. A reduction in s increases competition, reducing both prices and dispersion.

V. Conclusions

The paper explored the implications of a non-uniform costumers' distribution in a spatial location model. In a free entry model, the firms will be unequally distributed across regions if the costumers are. Larger regions have more firms, lower prices and lower average transportation costs than smaller regions. If we interpret the distribution of costumers as a simple wealth distribution, this exacerbates differences across regions, because poorer (less populated in the model) regions will have less firms each charging higher prices.

Additionally it studies the effect of reducing the transportation cost on the spatial location of firms. As in the uniform case, lower transportation costs produce more distance between contiguous firms, but lower prices. It also studies changes in the dispersion of individuals across regions. In this case, regions close enough to the mode of the distribution would face the lowest average distance and prices for small values of the dispersion parameter. However, this does not apply if we consider regions distant from the mode.

References

Anderson, S.P., Goeree, J.K. and Ramer, R. (1997). Location, Location, Location. In *Journal of Economic Theory*, *77*, pp. 102-27. Elsevier B.V.

D'Asprenmont, C., Gabszewicz, J.J., and Thisse, J-F (1979). On Hotelling's 'stability in competition'. In *Econometrica*, *47*(5), pp. 1145-1150. Malden (MA): John Wiley & Sons, Inc.

Dranove, D., Gron, A., and Mazzeo, M.J. (2003). Differentiation and competition in HMO markets. In *Journal of Industrial Economics*, *51*(4), pp. 433-454. Oxford (UK) and Malden (MA): Blackwell Publishing.

Eaton, B.C. (1976). Free entry in one-dimensional models: pure profits and multiple equilibria. In *Journal of Regional Science*, *16*(1), pp. 21-33. Malden (MA): John Wiley & Sons, Inc.

Economides, N.S. (1989). Symmetric equilibrium existence and optimality in differentiated product markets. In *Journal of Economic Theory*, *47*, pp. 178-94. Elsevier B.V.

Felici, R. and Pagnini, M. (2008). Distance, bank heterogeneity and entry in local banking markets. In *Journal of Industrial Economics*, *56*(3), pp. 500-534. Oxford (UK) and Malden (MA): Blackwell Publishing.

Gabszewicz, J.J., and Thisse, J-F. (1986). Spatial competition and the location of firms. In Gabszewicz, J.J., Thisse, J-F., Fushita, M. and Schweizer, U. (Eds): *Location theory*. Chur (Switzerland): Harwood Academic Publishers GmbH.

Hotelling, H. (1929). The stability of competition. In *Economic Journal*, *39*(153), pp. 41-57. Malden (MA): John Wiley & Sons, Inc. on behalf of Royal Economic Society.

Lewis, M. (2008). Price dispersion and competition with differentiated sellers. In *Journal of Industrial Economics*, *56*(3) 654-678. Oxford (UK) and Malden (MA): Blackwell Publishing.

Montes-Rojas, G. (2008). Non-uniform wealth distribution in a simple spatial banking model. In *Journal of Applied Economics*, *XI*(1), pp. 145-165. Buenos Aires (AR): Universidad del CEMA.

Neven, D.J. (1986). On Hotelling's competition with non-uniform customer distribution. In *Economics Letters*, *21*(2), pp. 121-26. Elsevier B.V.

Petersen, M.A., and Rajan, R.G. (2002). Does distance still matter? The information revolution in small business lending. In *Journal of Finance*, *57*(7), pp. 2533-70. Malden (MA): John Wiley & Sons, Inc.

Pilloff, S.J., and Rhoades, S.A. (2002). Structure and profitability in banking markets. In *Review of Industrial Organization*, *20*(1), pp. 81-98. Netherlands: Kluwer Academic Publishers.

Salop, S.C. (1979). Monopolistic competition with outside goods. In *Bell Journal of Economics*, *10*(1), pp. 141-156. Santa Monica (CA): The RAND Corporation.

Syverson, C. (2007). Prices, spatial competition and heterogeneous producers: An empirical test. In *Journal of Industrial Economics*, *55*(2), pp. 197-222. Oxford (UK) and Malden (MA): Blackwell Publishing.

Tabuchi, T., and Thisse, J-F. (1995). Asymmetric equilibria in spatial competition. In *International Journal of Industrial Organization*, *13*(2), pp. 213-27. Elsevier B.V.

Tirole, J. (1988). *The theory of industrial organization*. Cambridge (MA): MIT Press.

Appendix

Proof of Lemma 1: The existence and uniqueness of a Nash equilibrium has been extensively analyzed for the uniform case (see Gabszewicz and Thisse, 1986). The price differentiation set up and the assumption that individuals can only buy services from their nearest firms allows proving the existence and uniqueness of Nash equilibriums by looking at pair-wise Nash equilibriums of two firms each controlling only one price.

For the non-uniform case, it involves solving a two-nonlinear-equations and two unknowns system. Without loss of generality assume that the firms i=0 and i=1 are contiguous with locations $z_0 = 0$ and $z_1 = 1$ respectively. Assume that v is large enough to generate competition between the two firms (if that is not the case, the firms behave as monopolists, which generates a unique price strategy). Define $g_0(P_0^+, P_1^-)$ and $g_1(P_0^+, P_1^-)$ as the first order conditions of problem (6),

$$g_0(P_0^+, P_1^-) \equiv \int_0^{t(P_0^+, P_1^-)} f(x)dx - \frac{1}{2h}(P_0^+ - c) f\left(t(P_0^+, P_1^-)\right) = 0,$$

$$g_1(P_0^+, P_1^-) \equiv \int_{t(P_0^+, P_1^-)}^{1} f(x)dx + \frac{1}{2h}(P_1^- - c) f\left(t(P_0^+, P_1^-)\right) = 0,$$

where the two unknowns are (P_0^+, P_1^-) and

$$t(P_0^+, P_1^-) = \min\left(\max\left(\frac{-P_0^+ + P_1^- + h}{2h}, 0\right), 1\right)$$

Note that both functions are continuous in both arguments. Consider

$$\widetilde{P} = c + 2h \frac{\int_0^{1/2} f(x)}{f(1/2)}$$

and let $\overline{P}$ be a large enough price (i.e. v). Define $X_0^+ = \{(P_0^+, P_1^-) : P_0^+ \in [c, \widetilde{P})$, $P_1^- \in [\widetilde{P}, \overline{P})\}$ and $X_0^- = \{(P_0^+, P_1^-) : P_0^+ \in [\widetilde{P}, \overline{P})$, $P_1^- \in [c, \widetilde{P})\}$. Note that for any $(P_0^+, P_1^-) \in X_0^+$ we have $g_0(.) > 0$ while for $(P_0^+, P_1^-) \in X_0^-$, $g_0(.) \leq 0$. Also similar arguments can be used to show the existence of the sets X_1^+ and X_1^- which produce positive and non-positive values of $g_1(.)$ respectively. The existence of a solution follows from the existence of a zero in this nonlinear mapping, which is a generalization of the one-dimensional intermediate value theorem. Uniqueness is a consequence of the monotonicity of *f*(.).■

Proof of Lemma 2: In a uniform distribution, any potential new entrant (*k*) will locate at:

$$z_k = \frac{z_{j+1}^* + z_j^*}{2}$$

Moreover, prices will be symmetric (i.e. $P = P_k^+ = P_k^- = P_j^{+*} = P_{j+1}^{-*}$) and profits for firm *k* are a non-decreasing function of the distance $d = z_{j+1}^* - z_j^*$. Entry deterrence is obtained by finding *d* which satisfies

$$\frac{d}{2} + \left(-\frac{1}{2h}\right)(P - c) = 0,$$

$$\frac{d}{2}(P - c) = s,$$

where the first equation corresponds to the first order condition, taking into consideration the potential (symmetric) effect on their competitors and the second equation is the zero-profit condition. Solving for *d*, we get $d = \sqrt{2s / h}$. Therefore no additional entrances would occur for a location sequence such that:

$$\max\left\{z_j^* - z_{j-1}^*, z_{j+1}^* - z_j^*\right\} \le \sqrt{\frac{2s}{h}} \text{ for all } j.$$

In a maximum distance SPNE we have $z_j^* - z_{j-1}^* = \sqrt{2s / h}$ for all *j*, which generates a profit sequence $\{\Pi_j^* = s\}_{-\infty}^{\infty}$. A zero-profit SPNE is obtained by setting $z_j^* - z_{j-1}^* = \sqrt{s / h}$ for all *j*. Note that firms have no incentives to change location and symmetric distance, $z_j^* - z_{j-1}^* = z_{j+1}^* - z_j^*$, is optimal in the uniform case.■

Proof of Lemma 3: It is only needed to show that in the first stage, for fixed locations of the other firms, no firm has incentives to move. Consider firm j satisfying $z^*_{j-1} < z^*_j < z^*_{j+1}$ with profit level Π^*_j located in the non-uniform density region. Suppose that a new location is preferable (say $0 < z^*_{j-1} < z_j < z^*_j < z^*_{j+1}$, where the firm j prefers to move to a higher density zone). This motivates the entrance of at least one more firm (for more than one firm a similar argument applies). If only one firm enters (say k) it will be located to the right of the j-firm: $z^*_{j-1} < z_j < z_k < z^*_{j+1}$. If $z_k \leq z^*_j$, clearly the new location of firm j would generate negative profits (by the entry-deterrence condition), then $z_k > z^*_j$. Moreover by assumption, $\sup_z \Pi\left(z; z_j, z_k\right) \leq 0$ (only one additional firm). Then, if z_j becomes closer to z^*_{j-1}, profits became non-positive (since z_k goes to z^*_j), which implies that profits increase in the opposite direction. As z_j approaches z^*_j, profits are below to original level (since there is an additional firm k). The location compatible property implies that $\sup_z \Pi\left(z; z^*_{j-1}, \Delta(z, z^*_{j+1})\right) \leq \Pi^*_j$, therefore the profit level cannot be above the original one, which contradicts that the new location was better.■